LUND UNIVERSITY
Mechanics, LTH

Med lösningar

Mechanical Vibrations, FMEN11

2017-12-21, 14.00-19.00

Name (print):..
(or personal identifier)

Personal id-number:...................................
(or personal code)

Written examination with five examination tasks. Please check that all tasks are included. A clean copy of the solutions should be handed in. Write your name on all pages, number the pages and hand them in, in the correct order. Make sure that the papers are stapled together.

Summaries, equation sheets and mathematical handbooks such as TEFYMA may be used.

Results:

Task	Comment	Points (0-3)
1		
2		
3		
4		
5		
	Sum	
	Grade	

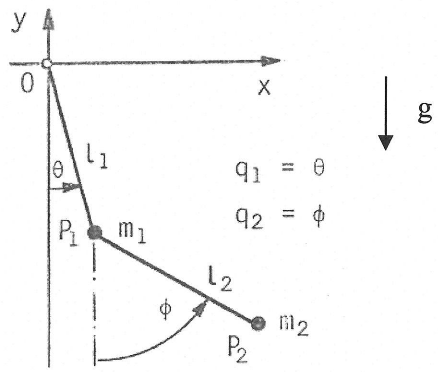
Name (signature):.. Leg:......
(Only if not anonymous)

1)

a) A double pendulum consists of two particles P_1 and P_2 with masses m_1 and m_2 respectively, connected by two strings of lengths l_1 and l_2 according to the figure below. The equations of motion for the system are:

$$a\ddot{\theta} + c\cos(\theta-\phi)\ddot{\phi} + c\sin(\theta-\phi)\dot{\phi}^2 + d\sin\theta = 0$$
$$b\ddot{\phi} + c\cos(\theta-\phi)\ddot{\theta} - c\sin(\theta-\phi)\dot{\theta}^2 + e\sin\phi = 0$$

Linearize the equations of motion at the equilibrium state $\theta = \phi = 0,\ \dot{\theta} = \dot{\phi} = 0$ and write on matrix format. Identify the mass matrix, the stiffness matrix, the damping matrix and the gyroscopic matrix.



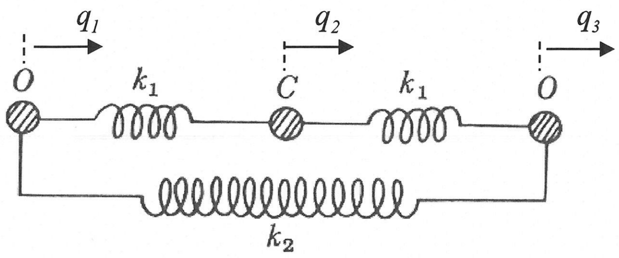
b) Let $q = q_1, \dots, q_n$ be generalized coordinates for a material system $\mathcal{B}$ subjected to the generalized forces $Q_i = \int_{\mathcal{B}} \boldsymbol{f}_a \cdot \frac{\partial \boldsymbol{r}}{\partial q_i} dm$, where $\boldsymbol{f}_a$ is the (specific) active accelerating force. The position vector of a material point $P \in \mathcal{B}$ is then given by $\boldsymbol{r} = \boldsymbol{r}(q;P)$. Let $\boldsymbol{a} = \boldsymbol{a}(q,\dot{q},\ddot{q};P)$ denote the acceleration of the material point.

Show that

$$\boldsymbol{a} = \sum_{j=1}^{n} \frac{\partial \boldsymbol{r}}{\partial q_j}\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\partial^2 \boldsymbol{r}}{\partial q_j \partial q_k}\dot{q}_j\dot{q}_k$$

c) During the site visit at Öresundsverket in Malmö, our hosts told us about different vibration phenomena that occur in their line of work. One example of vibrations that was discussed was axial vibrations in a turbine. Normally one would expect radial vibrations due to unbalance relative to the central shaft, but the manufacturers of the turbine had encountered problems with axial vibrations along the length of the turbine. What design change did they introduce in order to avoid these axial vibrations?

f) Consider linear vibrations of the CO_2 molecule (that is, vibrations in line with the molecule, see figure below). The mass of an oxygen atom is denoted by m_O and the mass of a carbon atom by m_C. The inter-atomic forces are represented by elastic springs with spring constants k_1 and k_2, respectively, according to the figure below.



Suppose that $x_1(0) = x_2(0) = x_3(0) = 0$, and that the masses are initially at rest, except for the carbon atom which is given the initial velocity 10m/s. What is the expression for the subsequent motion $\overline{q}(t)$ of the mechanical system when contributions equal to zero are omitted from the expression? Also identify the name of the different variables in the expression.

2)

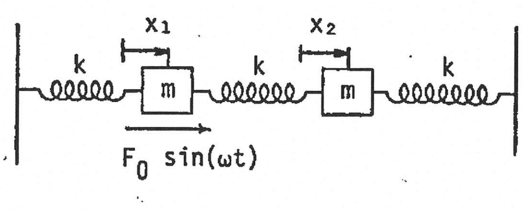
a) Consider the mechanical system in the figure below. The masses move along a fixed horizontal line and the left mass is subjected to an external harmonic force with amplitude F_0 and angular frequency ω. The mechanical system has the following mass and stiffness matrix

$$\underline{M} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \quad \underline{K} = \begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix}$$

and mode shapes

$$\overline{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \overline{x}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Calculate the modal mass matrix.



d) An n-dimensional mechanical system with positive definite mass matrix $\underline{M}$ and positive semi-definite stiffness matrix $\underline{K}$ has the eigenfrequencies $\omega_1, \omega_2, \ldots, \omega_n$ and the modal matrix $\underline{X} = [\overline{x}_1 \ldots \overline{x}_n]$ where

$$(-\omega_i^2 \underline{M} + \underline{K})\overline{x}_i = \overline{0}, \quad i = 1, \ldots, n$$

and

$$\underline{X}^T \underline{M}\,\underline{X} = \underline{\mu} = diag(\mu_1 \ldots \mu_n) \text{ and } \underline{X}^T \underline{K}\,\underline{X} = \underline{\kappa} = diag(\kappa_1 \ldots \kappa_n)$$

where $\mu_1, \ldots, \mu_n$ and $\kappa_1, \ldots, \kappa_n$ are constants satisfying $\mu_i > 0$ and $\kappa_i \geq 0$. The kinetic and potential energies of the system are given by

$$T = \frac{1}{2}\dot{\overline{q}}^T \underline{M}\dot{\overline{q}} \quad \text{and} \quad V = \frac{1}{2}\overline{q}^T \underline{K}\overline{q}$$

respectively. Using *normal coordinates* $\overline{\eta} = (\eta_1 \quad \eta_2 \quad \cdot \cdot \cdot \quad \eta_n)^T$ defined by $\overline{q} = \underline{X}\overline{\eta}$ Show that the equation for the free motion of the system may be written

$$\ddot{\overline{\eta}} + \underline{\omega}^2 \overline{\eta} = \overline{0}, \quad \text{where } \underline{\omega}^2 = \underline{\mu}^{-1}\underline{\kappa}$$

e) For vibrational motion of a bar in extension, L is the length of the bar, E is the modulus of elasticity, $A = A(x)$ is the sectional area, $k = k(x)$ is the external force density and $m = m(x)$ is the mass density. Assume that $k \equiv 0$ ("free vibrations") and consider a solution on the form ("separation of variables") $u(x,t) = \hat{u}(x)\phi(t)$, show that the following conditions are necessary, where λ is a constant.

$$-\frac{1}{m(x)}\frac{d}{dx}(EA(x)\frac{d\hat{u}(x)}{dx}) = \lambda \hat{u}(x) \quad \text{and} \quad \frac{d^2}{dt^2}\phi(t) + \lambda\phi(t) = 0$$

f) A mechanical system consists of ten masses $m_1, m_2, \ldots, m_{10}$ that are lined up after each other where each mass m_n is connected to the next mass $m_{(n+1)}$ using a spring with spring constant k_n, $n=1,2,\ldots,9$. Observe that mass m_1 is only connected to mass m_2 and that mass m_{10} is only connected to mass m_9. Calculations show that the system has the following natural eigenfrequencies:

$$\omega_{01} = 41.1,\ \omega_{02} = 79.4,\ \omega_{03} = 115.8,\ \omega_{04} = 157.3,\ \omega_{05} = 279.4,$$
$$\omega_{06} = 333.6,\ \omega_{07} = 577.9,\ \omega_{08} = 645.2,\ \omega_{09} = 792.5,\ \omega_{010} = 839.4$$

Referring to the numerical values of the eigenfrequencies, why is it reasonable to assume that the calculations are incorrect?

Introduce damping to the system by adding a viscous damper with damping coefficient c in between masses m_2 and m_3, in parallel with spring k_5.

a) Use coordinates x_1, x_2, x_3 to formulate the equations of motion for the system. (Introduction of mass matrices etcetera should be motivated using Lagrange´s equations, expressions of kinetic- and potential energy or free-body diagrams with accompanying equations of motion.)

b) Describe how to calculate the natural frequencies and their corresponding mode shapes, the damped natural frequencies and their corresponding mode shapes and the modal relative dampings.

c) Describe how to calculate the component $F_{22} = F_{22}(\omega)$ of the frequency response matrix.

In your description, please include the equations that have to be solved and present a list of names of all variables that are introduced in your equations. Necessary relationships that have to be used to solve the problem should be presented. Matlab and similar programs are just tools. Your task is to present a systematic description for the reader how to solve b) and c) using the results found in a).

5) In order to measure forces exerted by living nerve cells on their environment, nano beams are used where the deflections of the beams are measured. In order to calculate the force, physical properties of the nano beams need to be recorded. A first step is to formulate a mathematical model of the vibration of the nano beams. Formulate an expression for the mode shapes of transverse (bending) vibrations of a nano beam which is clamped (fixed) at one end and free at the other end. Also formulate an expression from which it is possible to numerically calculate the natural frequencies for the nano beams. The expressions may include mathematical functions such as for example sine and cosine, as well as the length of the nano wire: L, the sectional area: A, the sectional area moment-of-inertia: I, the material density: ρ and the modulus of elasticity: E. Assume that L, A, I, ρ, and E may be considered as constants. With given numerical values for the constants it should be possible to plot the theoretical mode shapes using your expression.