LUND UNIVERSITY
Mechanics, LTH

Mechanical Vibrations, FMEN10

2009-08-24, 14.00-19.00

Name (print):..

Personal id-number:...................................

Written examination with five examination tasks. Please check that all tasks are included. A clean copy of the solutions should be handed in. Write your name on all pages, number the pages and hand them in, in the correct order. Make sure that the papers are stapled together.

Summaries, equations-sheets, hand calculators and mathematical handbooks such as TEFYMA may be used.

Results:

Task	Comment	Points(0-3)
1		
2		
3		
4		
5		
	Sum	
	Grade	

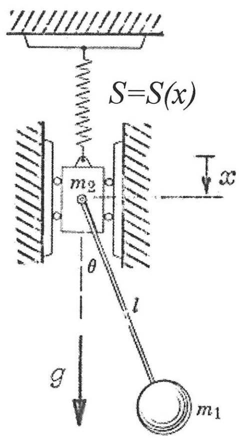
Name (signature):..

Leg:......

Term.räkn:

1)

A mechanical system consists of a particle pendulum with mass m_1 and cord-length l connected to a carriage, with mass m_2, which may move along a vertical guide, see figure below. The carriage is supported by a non-linear spring element with a retracting spring force equal to $S = S(x) = k_1 x + k_2 x^3$, $k_1, k_2 > 0$ where x is the elongation of the spring (measured from the unstressed state). The contact between the carriage and the guide walls gives rise to a constant frictional force $F_f > 0$ opposing the motion. Using the generalized coordinates x and θ, formulate the equations of motion for the system.



2)

A mechanical system is defined by its mass-matrix $\mathbf{M}$, its stiffness-matrix $\mathbf{K}$ and its damping matrix $\mathbf{C}$ where

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}, \quad \alpha, \beta \text{ (positive) real constants} \qquad (1)$$

The free vibrations of this system are given by the solution to the initial value problem:

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \\ &\mathbf{q}(0) = \mathbf{q}_0, \ \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0 \\ &\text{where } \mathbf{q}_0, \dot{\mathbf{q}}_0 \text{ are given constant vectors.} \end{aligned} \qquad (2)$$

Show that the solution to this problem is given by:

$$\begin{aligned} \mathbf{q}(t) = &\left[\sum_{i=1}^{n} e^{-\varsigma_i \omega_{0i} t} \left(\frac{\mathbf{x}_i \mathbf{x}_i^T \mathbf{M}}{\mu_i} \cos(\omega_{di} t) + \varsigma_i \frac{\omega_{0i}}{\omega_{di}} \frac{\mathbf{x}_i \mathbf{x}_i^T \mathbf{M}}{\mu_i} \sin(\omega_{di} t) \right) \right] \mathbf{q}_0 + \\ &+ \left[\sum_{i=1}^{n} e^{-\varsigma_i \omega_{0i} t} \frac{\mathbf{x}_i \mathbf{x}_i^T \mathbf{M}}{\mu_i \omega_{0i}} \sin(\omega_{di} t) \right] \dot{\mathbf{q}}_0 \end{aligned} \qquad (3)$$

where $\mathbf{x}_i$ denotes the classical mode shapes, $\mu_i, \omega_{0i}, \varsigma_i$ and ω_{di} are the modal mass, undamped natural frequency, relative damping and damped natural frequency respectively, corresponding to the i^{th} mode. We assume that $0 \le \varsigma_i < 1$ (weak damping).

3)

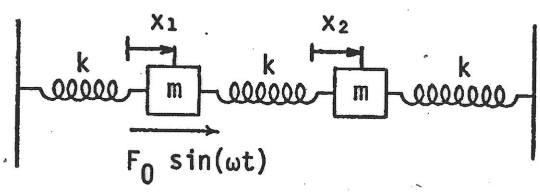
Consider the mechanical system in the figure below. The masses move along a fixed horizontal line and the left mass is subjected to an external harmonic force with amplitude F_0 and angular frequency ω. Introduce Rayleigh damping according to

$$\mathbf{C} = 0.006\mathbf{K} \qquad (1)$$

where **C** and **K** are damping- and stiffness-matrices respectively.

a. Use the coordinates x_1 and x_2 and formulate the equations of motion for the system.
b. Describe how to calculate the un-damped natural frequencies and the corresponding mode shapes. Also describe how to calculate the modal relative dampings, the damped natural frequencies and the corresponding mode shapes. Assume free vibrations ($F_0 = 0$). No numerical values need to be presented.
c. Describe how to calculate the forced vibration ($F_0 = 100\text{N}$) for the frequency: $\omega = 30\text{rad/s}$. Also assume that we are starting from rest, which means that $x_1(0) = x_2(0) = 0,\ \dot{x}_1(0) = \dot{x}_2(0) = 0$. No numerical values need to be presented.

In your description, please include the equations that have to be solved and present a list of names of all variables that are introduced in your equations. Necessary relationships that have to be used to solve the problem should be presented. Matlab and similar programmes are just tools. Your task is to present a systematic description to the reader how to solve b) and c) using the results found in a).



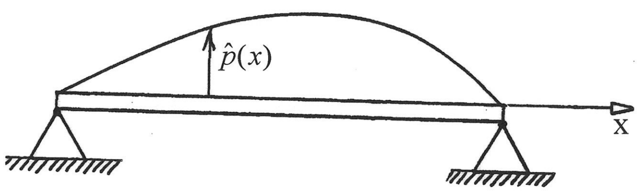
4)

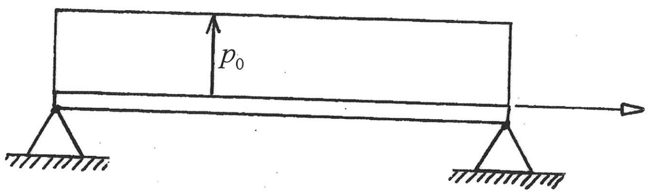
The simply supported beam, with density = ρ, modulus of elasticity = E, length = L, sectional area = A and sectional area moment-of-inertia = I (assumed to be constant), is subjected to an external transverse force

$$p = p(x,t) = \hat{p}(x)\sin \omega t \qquad (1)$$

Calculate the forced "steady-state" (only consider the particulate solution, assume that the homogeneous solution is damped out in the long run) response of the midpoint $x = \frac{L}{2}$ of the beam.

$\hat{p}(x) = p_0 = const.$





5)

The homogeneous beam has the bending stiffness EI, the mass m and the length L. Describe how to calculate the natural frequencies for the system in *transversal vibrations* using the FEA with a two-element approximation. The final step of the numerical calculation may be excluded from the solution.