MECHANICAL VIBRATIONS - Examination task 3

Name: ………………………………..

Problem 4:1

A homogeneous beam with length L_0 is clamped to a wall at one end and transversely supported by a linear elastic spring at the other according to the figure below. The beam has the mass m_0 per unit length, modulus of elasticity E and a sectional area moment-of-inertia I_0. The spring constant = k. The transversal deflection of the beam (in the z-direction), in a free vibration, is denoted $w = w(x,t),\ \ 0 < x < L_0$.

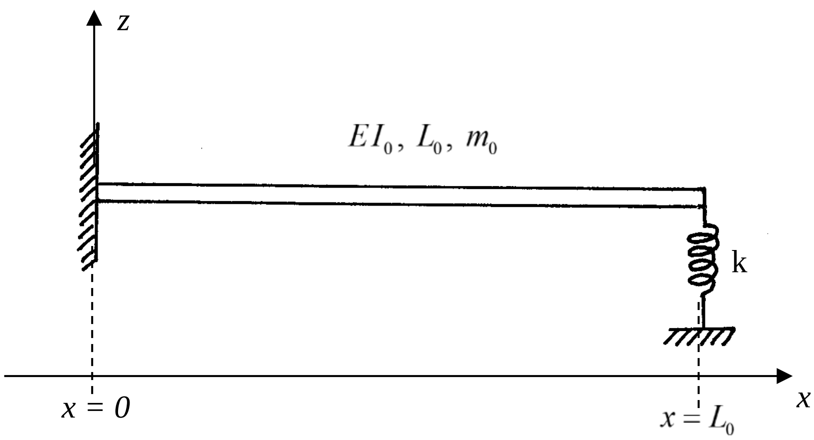
a) Consider a solution for the deflection on the form ('standing wave solution') $w(x,t) = \hat{w}(x)\phi(t)$ and derive the necessary conditions (differential equations, eigenvalue problems) that have to be fulfilled by the functions $\hat{w} = \hat{w}(x)$ and $\phi = \phi(t)$.

b) Formulate the boundary conditions (at $x = 0$ and $x = L_0$) for the 'mode-shape' $\hat{w} = \hat{w}(x)$.

c) Show that mode shapes $\hat{w}_1 = \hat{w}_1(x),\ \hat{w}_2 = \hat{w}_2(x)$ corresponding to different eigen-frequencies are orthogonal in the sense that

$$\int_0^L \hat{w}_1(x)\hat{w}_2(x)dx = 0 \qquad (1)$$

(Hint: Consult Exercise 5:2)

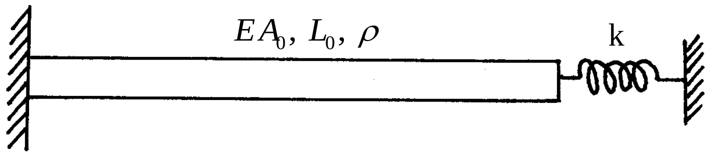


Solution:

Name:..

Problem 4:2

A homogeneous bar with length $L_0 = 2.0\text{m}$ is clamped to a wall at one end and longitudinally supported by a linear elastic spring at the other according to the figure below. The material of the bar has mass density $\rho = 7800\,\text{kgm}^{-3}$, modulus of elasticity $E = 200\,\text{GPa}$ The sectional area of the bar is $A_0 = 3\cdot 10^{-6}\text{m}^2$. The spring constant $k = 2\cdot 10^5\,\text{Nm}^{-1}$. Determine the *lowest* natural frequency of the longitudinal motion of the bar-spring system.



Solution:

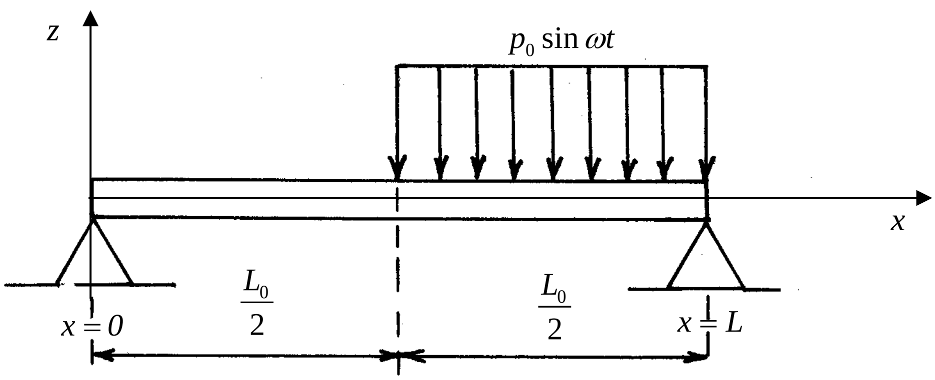
Name:...

Problem 4:3

A homogeneous simply supported beam with length $L_0 = 2.0\,\text{m}$ is subjected to an external distributed transversal force (see figure below) defined by

$$p = p(x,t) = \begin{cases} 0 & 0 \le x < \dfrac{L_0}{2} \\ p_0 \sin \omega t & \dfrac{L_0}{2} \le x \le L_0 \end{cases} \qquad (2)$$

where the amplitude of the force $p_0 = 1000\text{Nm}^{-1}$ and the angular frequency $\omega = 500\text{rads}^{-1}$. The beam has mass density $\rho = 7800\text{kgm}^{-3}$, modulus of elasticity $E = 200\,\text{GPa}$ and a squared cross sectional area with area $A_0 = 4.0 \cdot 10^{-4}\,\text{m}^2$ and area moment-of-inertia $I_0 = 1.7 \cdot 10^{-7}\,\text{m}^4$. Calculate the forced ('steady state') response of the system, in terms of transversal vibrations $w = w(x,t),\ \ 0 \le x \le L_0,\ \ t \ge 0$, when the beam is at rest with no deflection ($w = 0$) at time t = 0. Plot, (using Matlab, Mathcad, Maple, ...), the mid-beam deflection $w = w(\frac{L_0}{2}, t)$ in the time interval $0 \le t \le 0.1s$.



Solution:

Name:...

Problem 5:1

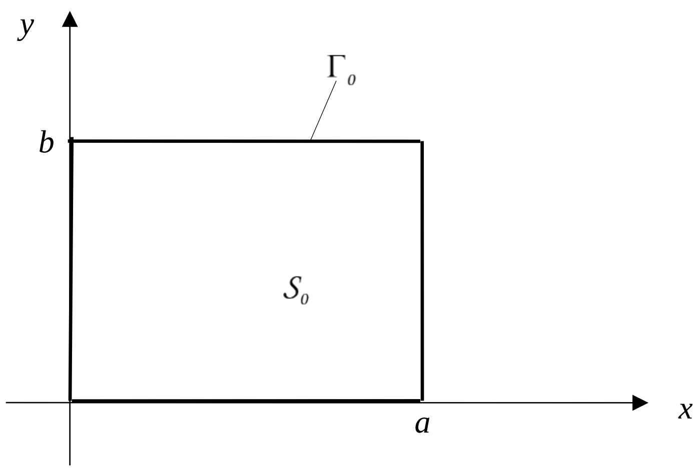
A uniform rectangular *membrane* with *mass per unit area* ρ_a is extending, in its un-deformed configuration, over a domain S_0 in the *x*-*y*-plane defined by

$$S_0 = \left\{(x,y) \in \mathbb{R}^2 : 0 \le x \le a,\ 0 \le y \le b\right\} \qquad (1)$$

The boundary of S_0 is the curve Γ_0 consisting of the straight line segments $x = 0,\ x = a$ and $y = 0,\ y = b$. The membrane is fixed along Γ_0 and the *pre-surface-tension* of the membrane is a constant equal to T_0. The membrane is subjected to an evenly distributed external force density in the z-direction:

$$p(x,y,t) = p_0 \sin \omega t, \quad (x,y) \in S_0, \quad t \ge 0 \qquad (2)$$

where p_0 is a given constant amplitude and ω a given angular frequency.
Calculate the forced ("steady state") transversal response of the system when the membrane is at rest with no deflection at time *t* = *0*.



Solution:

Name:..

Problem 5:2

A uniform rectangular *plate* with *mass per unit area* ρ_a and thickness *h* is extending, in its un-deformed configuration, over a domain S_0 in the x-y-plane defined by

$$S_0 = \{(x, y) \in \mathbb{R}^2 : 0 \le x \le a,\ 0 \le y \le b\} \qquad (1)$$

The boundary of S_0 is the "curve" Γ_0 consisting of the straight line segments $x = 0,\ x = a$ and $y = 0,\ y = b$. See figure in Problem 5:1. The plate is *simply supported* along Γ_0. The Young's modulus and the Poisson´s ratio of the plate material are denoted *E* and ν, respectively. The plate is subjected to a concentrated ("point") force in the z-direction

$$p(x, y, t) = P_0\, \delta(x - 0.75a, y - 0.5b) \sin \omega t, \quad (x, y) \in S_0,\ \ t \ge 0 \qquad (2)$$

where P_0 is a given constant force, ω a given angular frequency and $\delta = \delta(x - 0.75a, y - 0.5b)$ is the 'Dirac-function' localized to $x = 0.75a,\ y = 0.5b$, i.e. for any continuous function $f = f(x, y)$,

$$\iint_{S_0} f(x, y)\, \delta(x - 0.75a, y - 0.5b)\, dxdy = f(0.75a, 0.5b) \qquad (3)$$

Calculate the forced ("steady state") transversal response of the system when the plate is at rest with no deflection at time *t = 0*. Plot the motion of the centre of the plate ($x = \frac{a}{2},\ y = \frac{b}{2}$) during the time-interval $0 \le t \le 0.1\,s$ in a diagram (using Matlab, Mathcad, Maple, …).

Use the following data:

$\rho_a = 78\,\text{kgm}^{-2}$, $a = 1.0\,\text{m},\ b = 2.0\,\text{m},\ h = 10\text{mm}$

$E = 210\,\text{GPa},\ \nu = 0.3, P_0 = 10000\,\text{N},\ \omega = 400\,\text{rads}^{-1}$

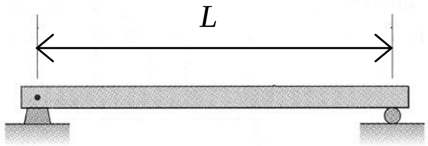
Solution:

Name:...

Problem 5:3

A homogeneous beam with *the total mass* m, length L and bending stiffness EI is *simply supported* at both its ends. Calculate the natural frequencies for the beam in *transversal vibrations* using the finite element approximation with

a) one element
b) two elements



Compare the natural frequencies of the two approximations with the exact frequencies!

Use the following data: $EI = 9.4 \cdot 10^5\,\text{Nm}^2$, $m = 10\,\text{kg}$, $L = 2.0\,\text{m}$.

Solution: