MULTIBODY DYNAMICS - Examination task 1

Name: ……..

Problem 1:1

A body $\mathcal{B}$ is subjected to two rotations represented by rotation tensors $\boldsymbol{R}_1 \sim \boldsymbol{n}_1\varphi_1$ and $\boldsymbol{R}_2 \sim \boldsymbol{n}_2\varphi_2$, respectively where, in an RON-basis $\underline{\boldsymbol{e}} = (\boldsymbol{e}_1 \;\; \boldsymbol{e}_2 \;\; \boldsymbol{e}_3)$; $\boldsymbol{n}_1 = \underline{\boldsymbol{e}}\left[\boldsymbol{n}_1\right]_{\underline{\boldsymbol{e}}}$, $\boldsymbol{n}_2 = \underline{\boldsymbol{e}}\left[\boldsymbol{n}_2\right]_{\underline{\boldsymbol{e}}}$ and

$$\left[\boldsymbol{n}_1\right]_{\underline{\boldsymbol{e}}} = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}, \;\; \varphi_1 = \frac{\pi}{2} \;\; \text{and} \;\; \left[\boldsymbol{n}_2\right]_{\underline{\boldsymbol{e}}} = \begin{pmatrix}0\\1\\0\end{pmatrix}, \;\; \varphi_2 = \frac{\pi}{6} \qquad (1)$$

a) Calculate $\boldsymbol{R}_2\boldsymbol{R}_1$
b) Calculate the Euler parameters e_0, e_1, e_2, e_3 corresponding to $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively.
c) Calculate the Euler parameters corresponding to $\boldsymbol{R}_2\boldsymbol{R}_1$ by using Proposition 4.1(p. 69) in the Lecture Notes.
d) Use the result in c) to calculate $\boldsymbol{n}$ and φ so that $\boldsymbol{R}_2\boldsymbol{R}_1 \sim \boldsymbol{n}\varphi$.
e) Use the Matlab function **eig** to calculate $\boldsymbol{n}$ and φ so that $\boldsymbol{R}_2\boldsymbol{R}_1 \sim \boldsymbol{n}\varphi$.

Solution:

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Problem 1:2

Given a tensor $\boldsymbol{R}$ with the matrix

$$[\boldsymbol{R}]_{\underline{e}} = \begin{pmatrix} 0.9082 & -0.3994 & -0.1247 \\ 0.4082 & 0.9112 & 0.0547 \\ 0.0917 & -0.1007 & 0.9906 \end{pmatrix} \tag{2}$$

where $\underline{\boldsymbol{e}} = (\boldsymbol{e}_1 \;\; \boldsymbol{e}_2 \;\; \boldsymbol{e}_3)$ is a RON-basis.

a) Show that $\boldsymbol{R}$ is a rotation tensor (approximately).
b) Calculate the rotation vector $\boldsymbol{n}\varphi$
c) Calculate the tensor $\boldsymbol{N} = \boldsymbol{n} \times \boldsymbol{1}$
d) Calculate, using Matlab, $\exp(\varphi \boldsymbol{N})$.

Solution:

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Problem 1:3

The thin circular wheel $\mathcal{W}$, of radius $r = 0.5\,m$, rolls in a circle of radius $b = 1.5m$ on the disk *C* which is *fixed to ground*. The axle *OA* rotates about the vertical axis $(O, \boldsymbol{e}_3^o)$ at the constant angular velocity $\omega_0 = \frac{\pi}{2} rad\, s^{-1}$ where $\underline{\boldsymbol{e}}^o = (\boldsymbol{e}_1^o \;\; \boldsymbol{e}_2^o \;\; \boldsymbol{e}_3^o)$ is a RON-basis fixed to the disc *C*. Let $\underline{\boldsymbol{e}} = \underline{\boldsymbol{e}}(t) = (\boldsymbol{e}_1(t) \;\; \boldsymbol{e}_2(t) \;\; \boldsymbol{e}_3(t))$ be a RON-basis *fixed to the wheel* $\mathcal{W}$ where $\boldsymbol{e}_1$ is directed along *OA* and perpendicular to the wheel, see the figure below! Let $\boldsymbol{R} = \boldsymbol{R}(t)$ denote the rotation tensor for $\mathcal{W}$, i.e.

$$\underline{\boldsymbol{e}}(t) = \boldsymbol{R}(t)\underline{\boldsymbol{e}}^0 \tag{3}$$

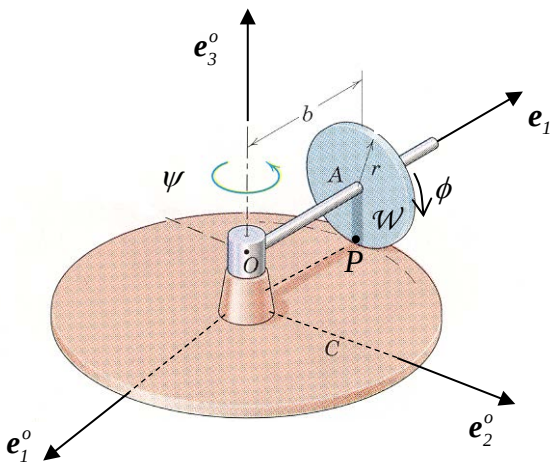
Assume that $\boldsymbol{R}(0) = \boldsymbol{1}$ so that $\underline{\boldsymbol{e}}(0) = \underline{\boldsymbol{e}}^o$.

a) Let ψ denote the rotation angle of the axle *OA* relative to the disc *C* and let ϕ denote the rotation angle of the wheel $\mathcal{W}$ relative to the axle *OA*. Express the rotation matrix $[\boldsymbol{R}]_{\underline{\boldsymbol{e}}^o}$ in terms of ψ and ϕ.

b) Calculate the matrix $[\boldsymbol{W}]_{\underline{\boldsymbol{e}}^o}$ for the spin tensor $\boldsymbol{W} = \dot{\boldsymbol{R}}\boldsymbol{R}^T$ in terms of ψ, ϕ, $\dot{\psi}$ and $\dot{\phi}$. Calculate the matrix $[\boldsymbol{\omega}]_{\underline{\boldsymbol{e}}^o}$ for the angular velocity, $\boldsymbol{\omega} = ax(\boldsymbol{W})$.

c) Calculate the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$ of the wheel $\mathcal{W}$ if it is *rolling without slipping* at its contact point with the disc *C*. Rolling without slipping means that $\boldsymbol{v}_P = \boldsymbol{0}$ where P is the material point in the wheel instantaneously in contact with the disc.

d) With presumptions as in c) calculate the matrices $[\boldsymbol{R}(t)]_{\underline{\boldsymbol{e}}^o}$, $[\boldsymbol{e}_1(t)]_{\underline{\boldsymbol{e}}^o}$ and $[\boldsymbol{e}_2(t)]_{\underline{\boldsymbol{e}}^o}$ at time $t = 2s$.



Solution: