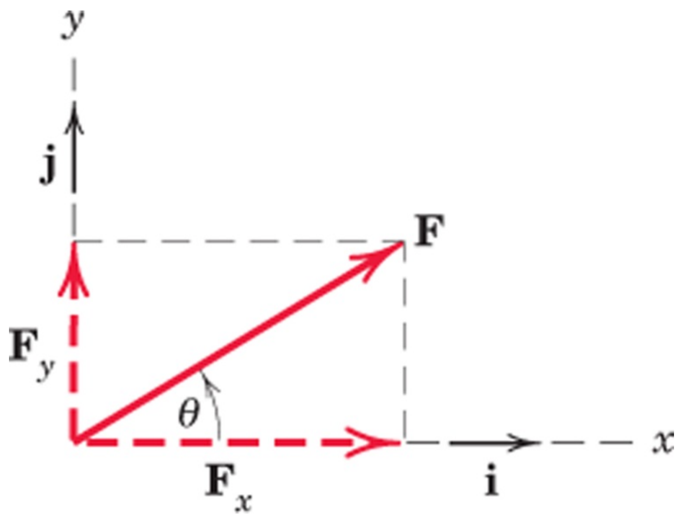
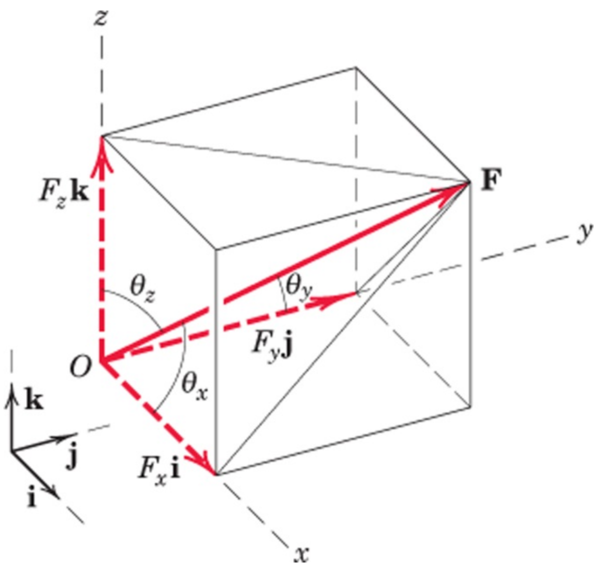
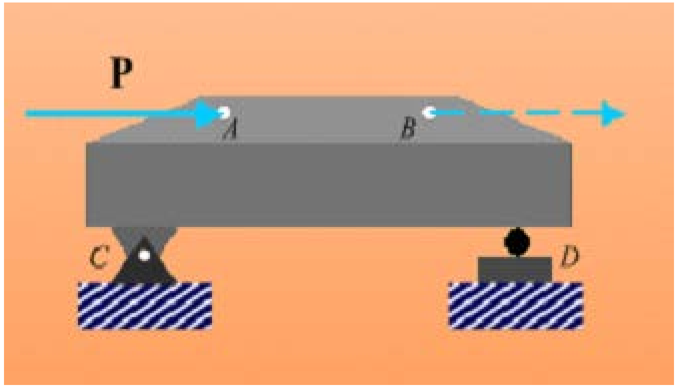
Repetition Mekanik, grundkurs

Kraft är en vektor och beskrivs med storlek riktning och angreppspunkt



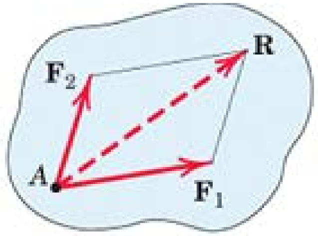


$$\mathbf{F} = F_x \mathbf{e}_x + F_y \mathbf{e}_y + F_z \mathbf{e}_z$$

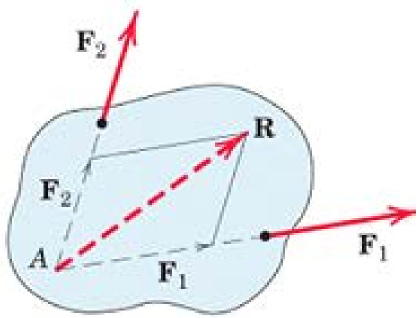


Kraften kan flytta längs sin verkninglinje

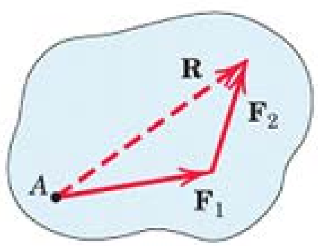
Addera krafter



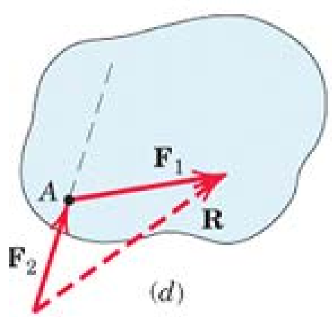
(a)



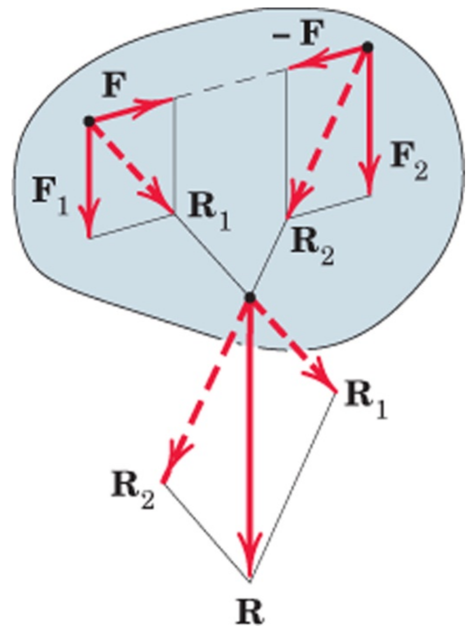
(b)



(c)

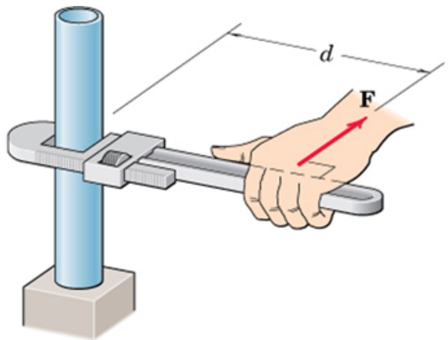


(d)

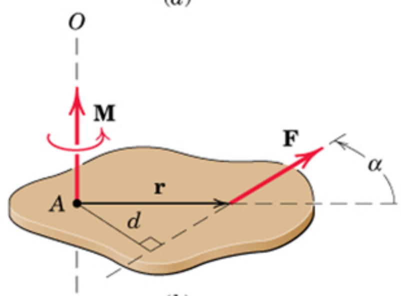


Moment i planet

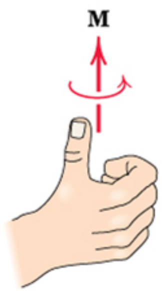
Kraftmoment-Kraftens vridande förmåga



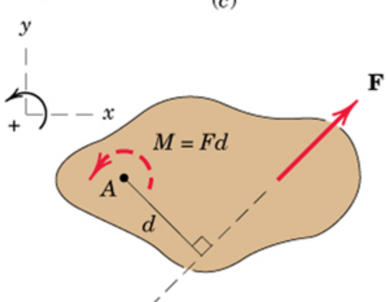
(a)



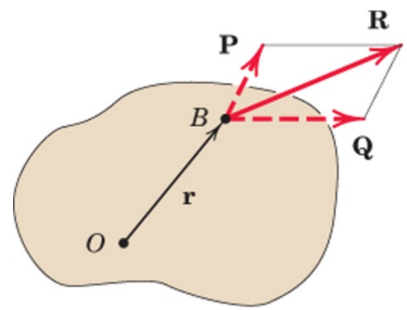
(b)



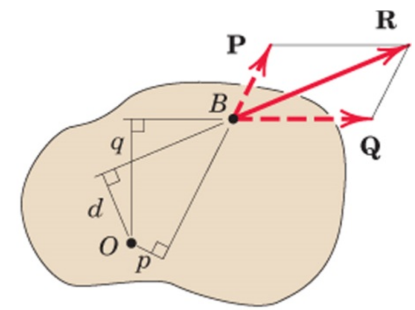
(c)



(d)



(a)

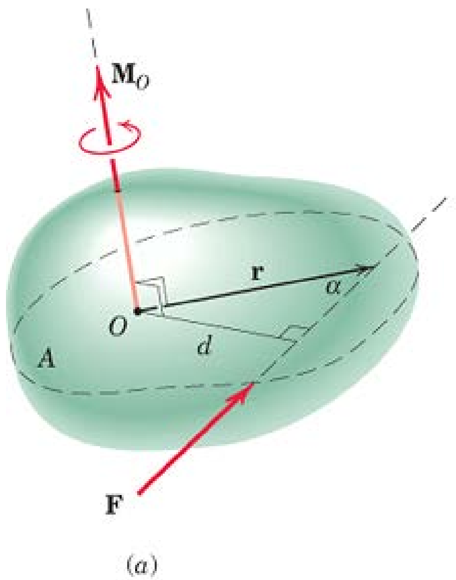


(b)

$$M_o = Rd = Pp + Qq$$

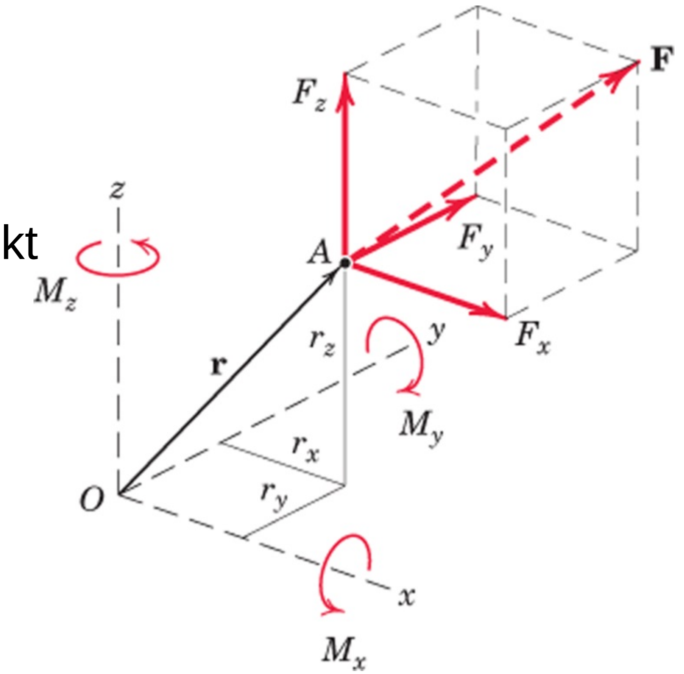
$$\mathbf{M}_0 = \mathbf{r} \times \mathbf{F}$$

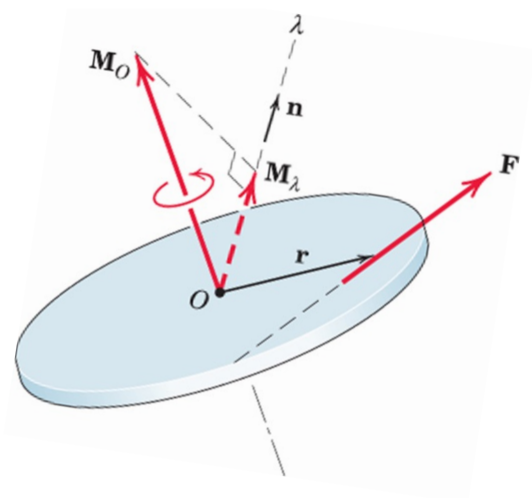
Kraftmoment i 3D



Kraftmoment m a p en punkt

$$\mathbf{M}_0 = \mathbf{r} \times \mathbf{F}$$



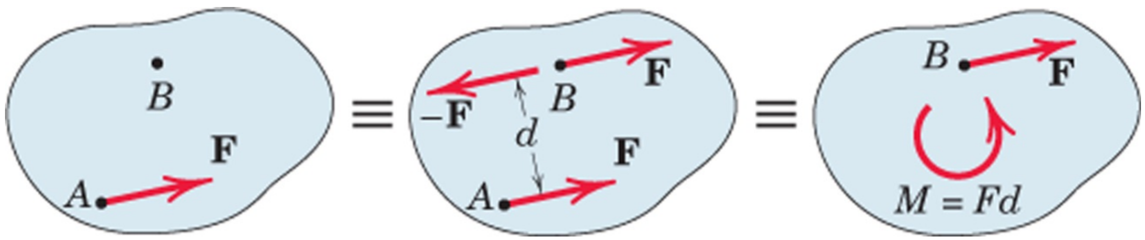


Kraftmoment m a p en axel

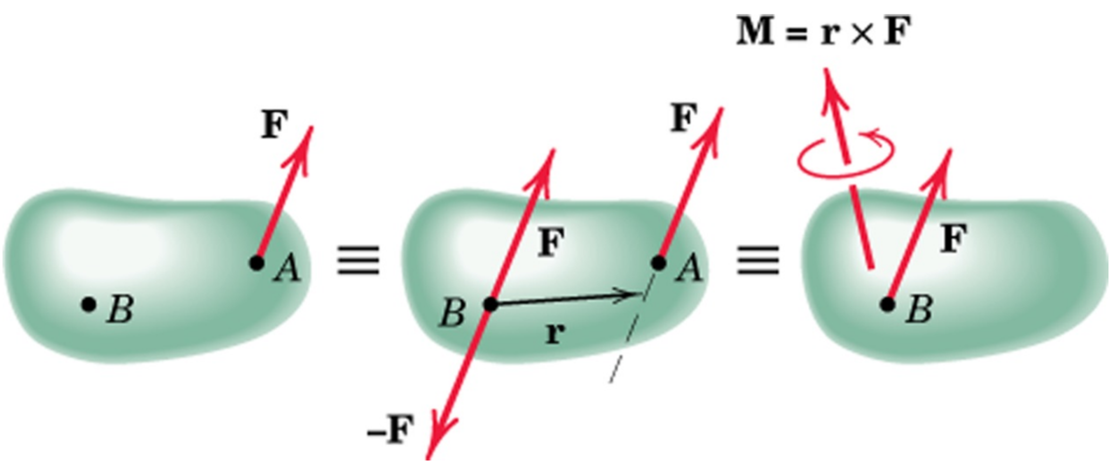
$$\mathbf{M}_\lambda = (\mathbf{M}_0 \cdot \mathbf{n})\mathbf{n} = ((\mathbf{r} \times \mathbf{F}) \cdot \mathbf{n})\mathbf{n}$$

Kraftpar

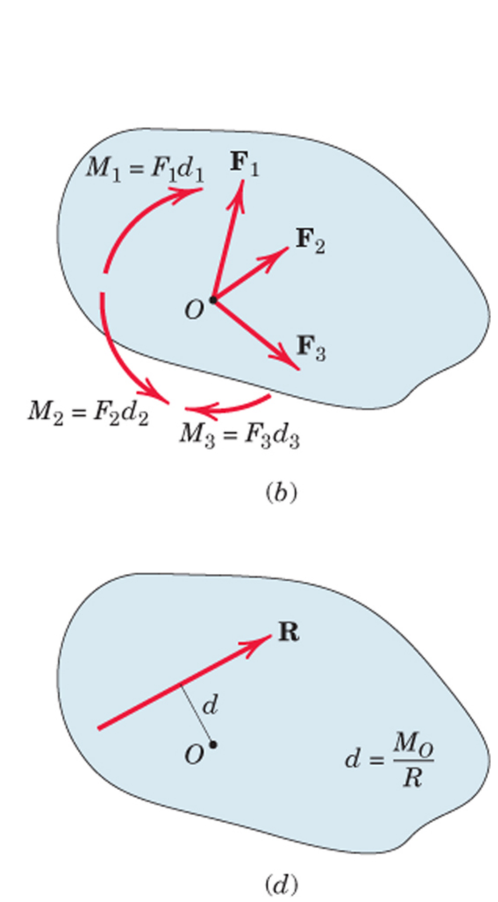
I planet



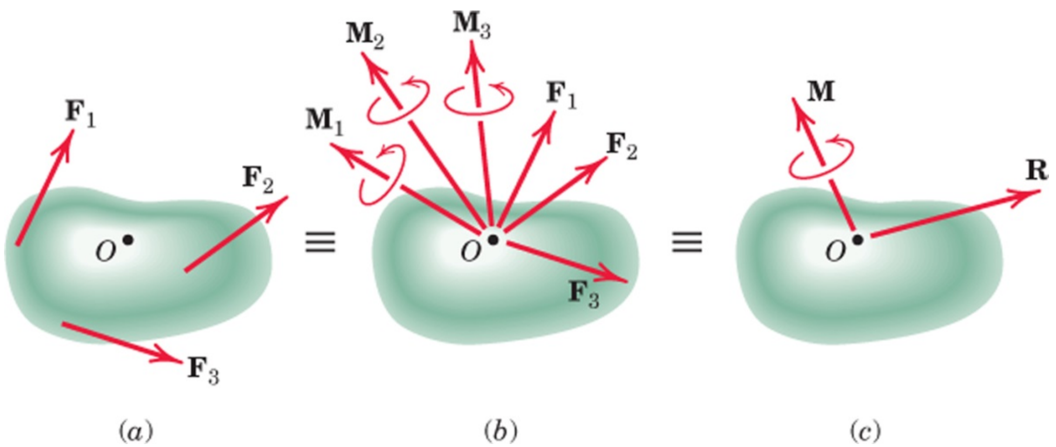
I 3D



Resultant i planet

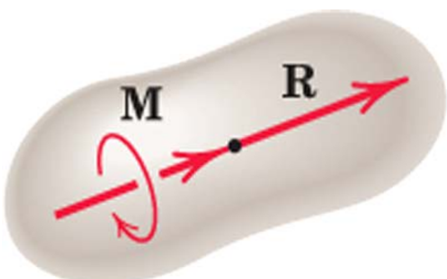


Resultant i 3D

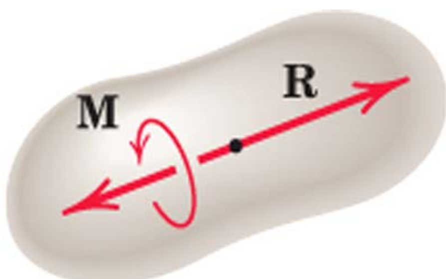


$$\mathbf{R} = \sum_i \mathbf{F}_i$$

$$\mathbf{M} = \sum_i \mathbf{M}_i$$



Positiv kraftskruv



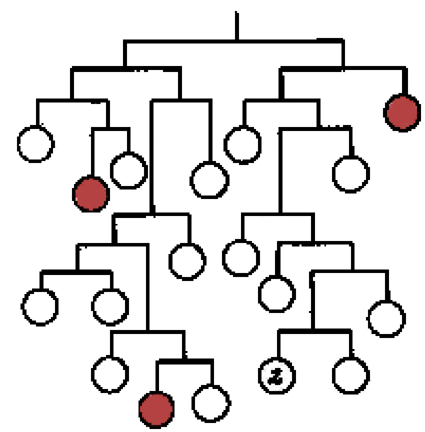
negativ kraftskruv

JÄMVIKT

$$\sum_i \mathbf{F_i} = 0$$

$$\sum_i \mathbf{M}_i = 0$$

=



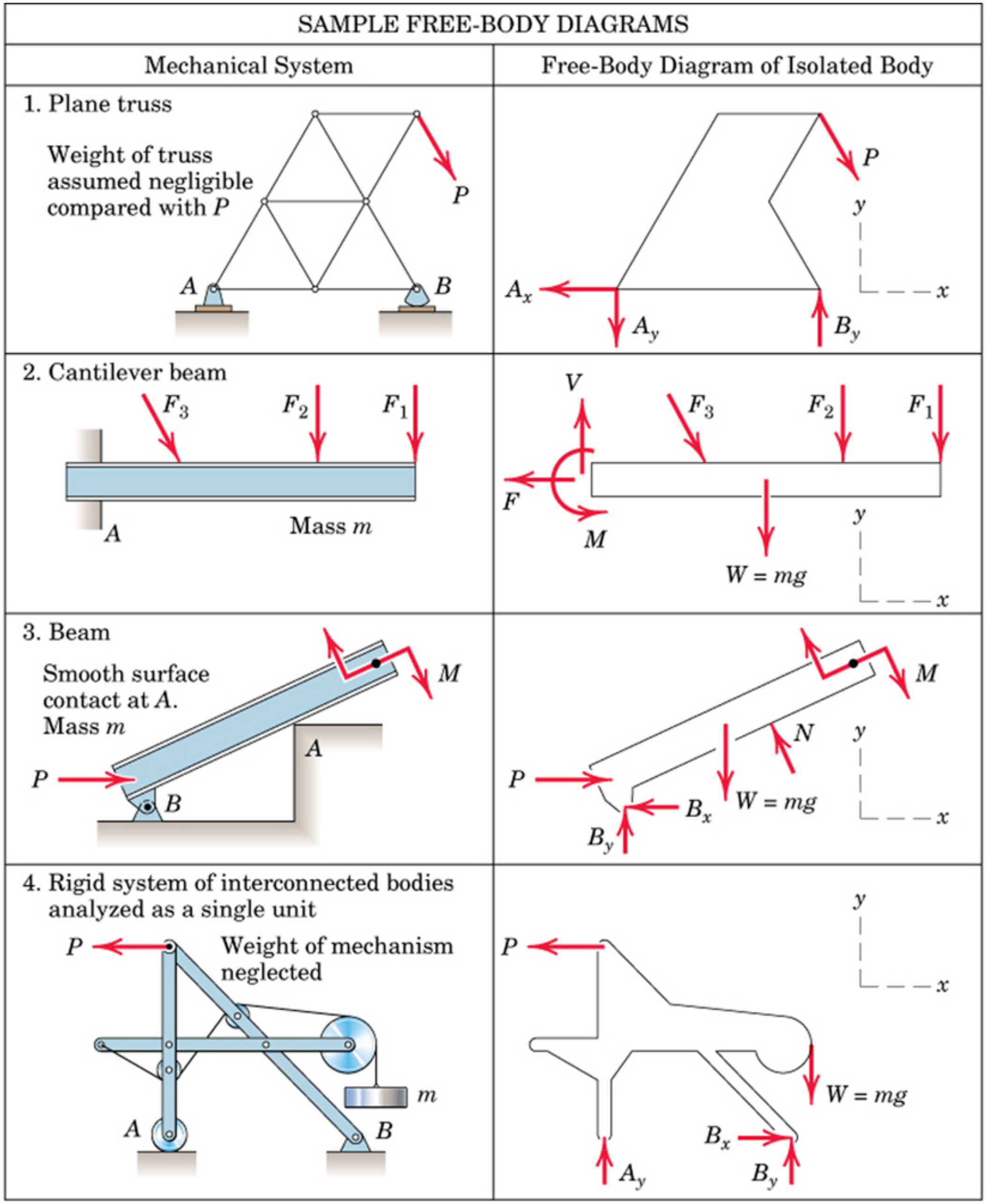
Friläggning

1. Fatta beslut om vilken eller vilka delar som ska friläggas

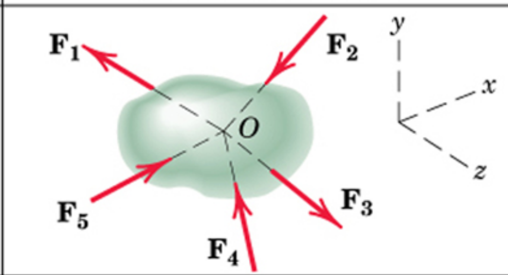
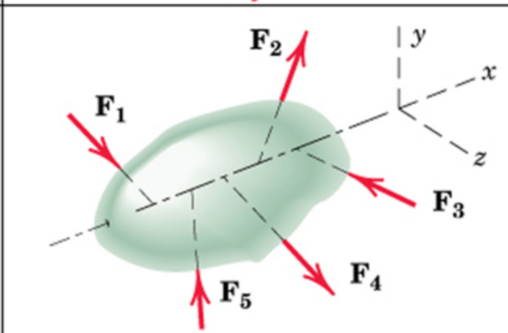
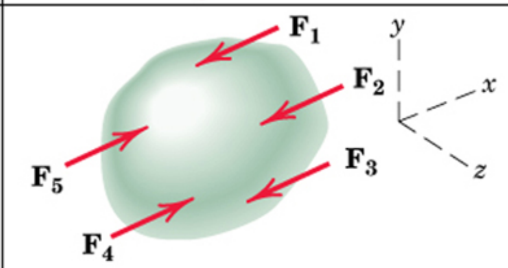
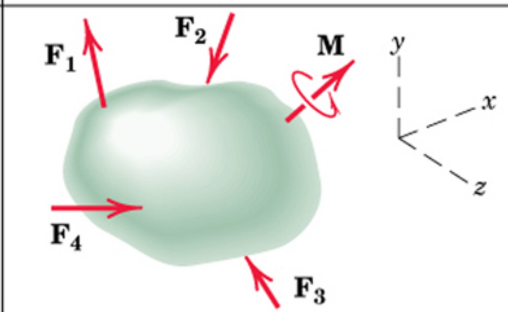
2. Rita en figur av den frilagda kroppen. (Kroppen ska vara totalt isolerad från omgivningen)

3.Sätt ut alla yttre krafter som verkar på kroppen inklusive tyngdkraft och alla ev. andra fältkrafter. Gå igenom ytterkonturen och sätt ut krafter och moment i de punkter där kroppen skurits av från omgivningen.

4. Lägg in ett lämpligt koordinatsystem i figuren.



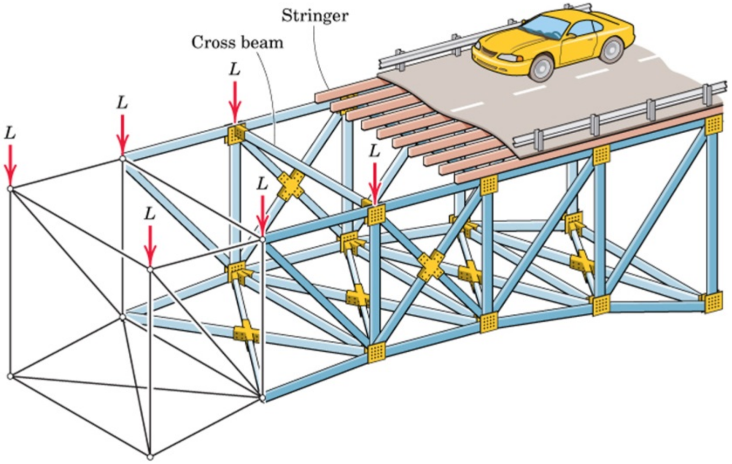
Jämvikt i 3D

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_x = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_z = 0$ $\quad \Sigma M_z = 0$

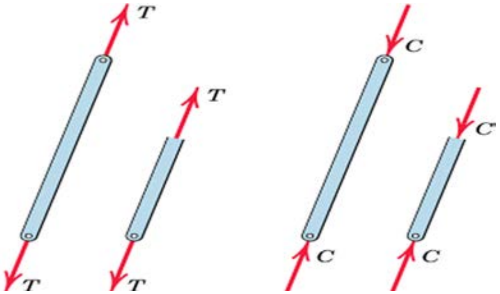
I 2D kan man ställa upp högst 3 oberoende jämviktsekvationer

I 3D kan man ställa upp högst 6 oberoende jämviktsekvationer

Fackverk



Tvåkraftsystem

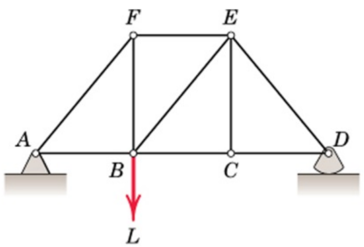


Tryck

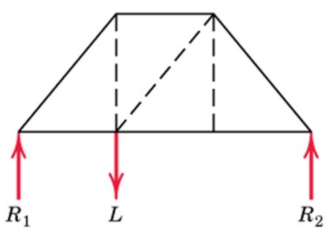
Drag

Knutpunktsmetoden

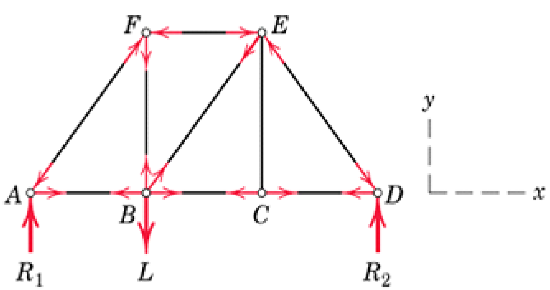
Steg1 : Frilläg hela systemet och bestäm reaktionskrafterna
Steg 2 : Gå systematiskt igenom alla knutpunkret och bestäm krafterna genom att ställa upp kraftekvationer i två riktningar
Steg 3: Avgör om det är tryck eller drag i stängerna



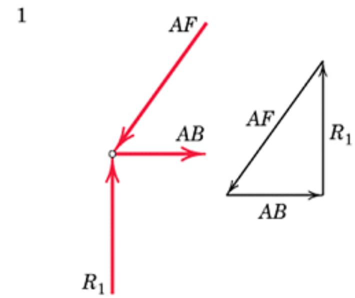
(a)



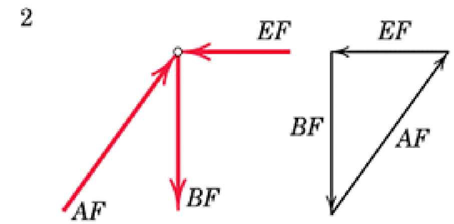
(b)



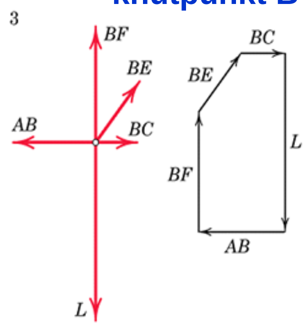
knutpunkt A



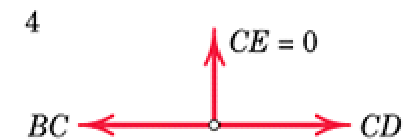
knutpunkt F



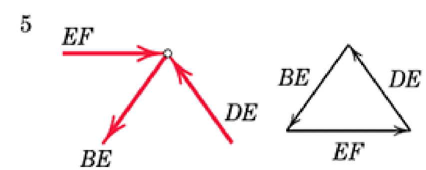
knutpunkt B



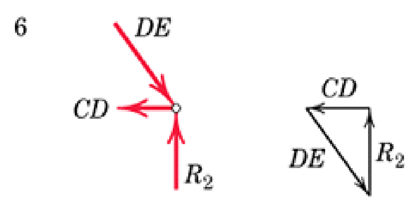
knutpunkt C



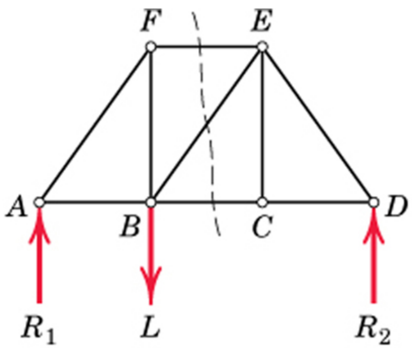
knutpunkt E



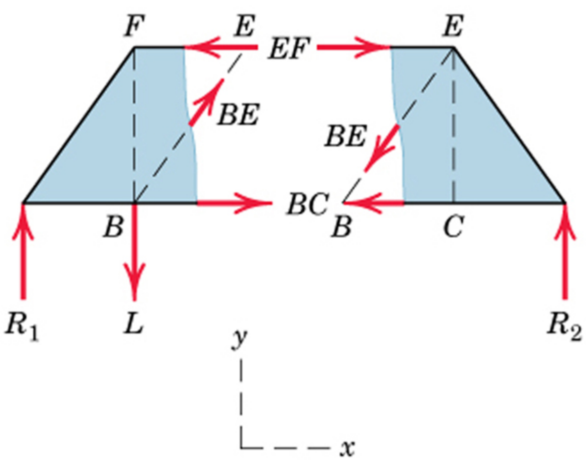
knutpunkt D



Fackverk



(a)

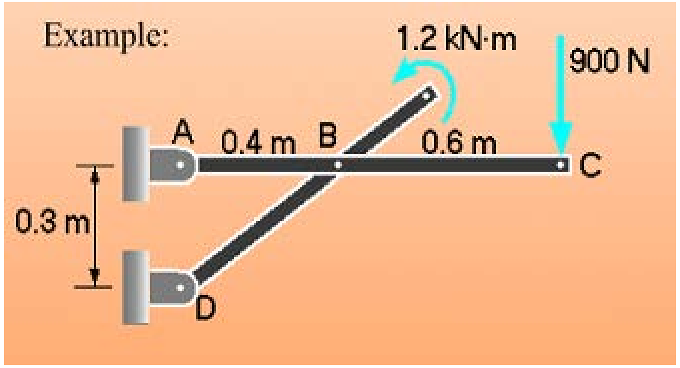


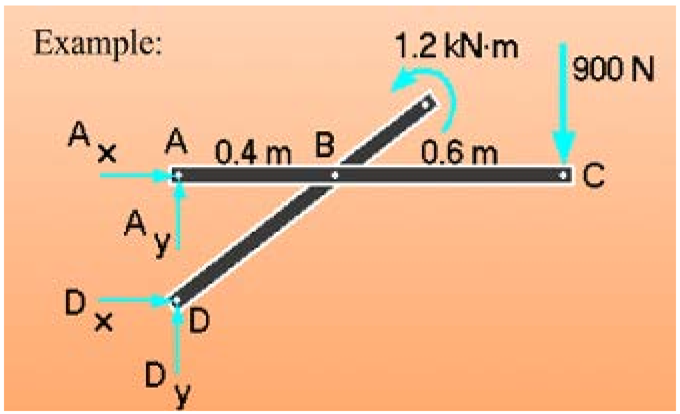
(b)

Snittmetoden

Steg1 : Friläg hela systemet och bestäm reaktionskrafterna
Steg 2 : Välj ut ett en av systemet och gör ett snitt.
Rita ut alla krafter i stängerna som snittades.
Steg 3 : Bestäm krafterna genom att ställa upp kraft- och
momentekvationer (högst 3 oberoende ekv.)
Steg 3: Avgör om det är tryck eller drag i stängerna

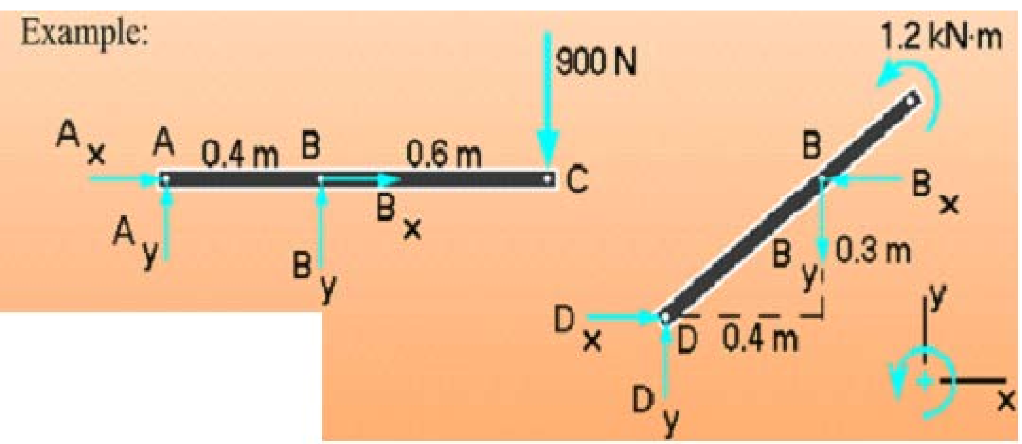
Strukturer





Steg1 : Frilägg hela systemet. Om man har fler än tre obekanta går det inte att räkna ut alla krafter.

Steg 2 : Frilägg varje del för sig och bestäm krafterna genom att ställa upp kraft- och momentekvationer.



Frame AC:

$$\Sigma F_x = 0 : \; A_x + B_x = 0$$

$$\Sigma F_y = 0 : \; A_y + B_y - 900 = 0$$

$$\Sigma M_A = 0 : B_y(0.4) - 900(1) = 0$$

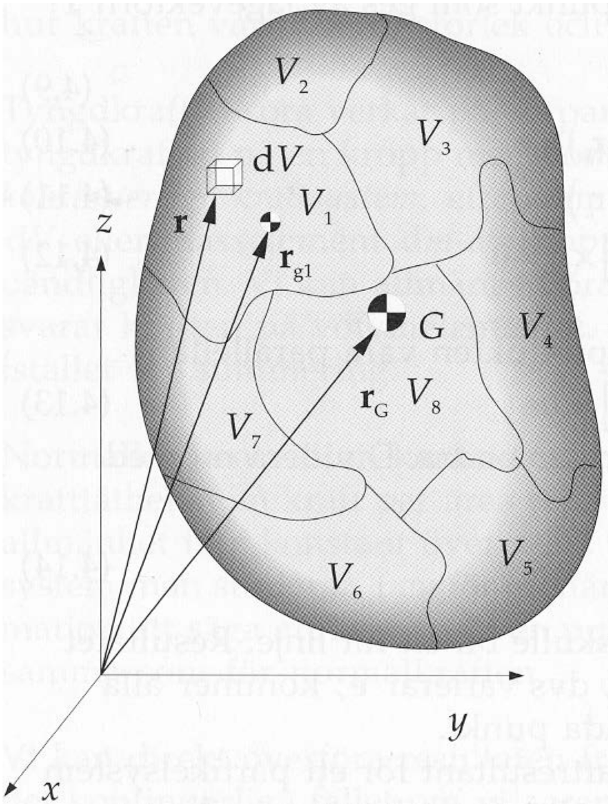
Frame BD:

$$\Sigma F_x = 0 : \; D_x - B_x = 0$$

$$\Sigma F_y = 0 : \; D_y - B_y = 0$$

$$\Sigma M_B = 0 : D_x\,(0.3) - D_y(0.4) + 1200 = 0$$

Masscentrum

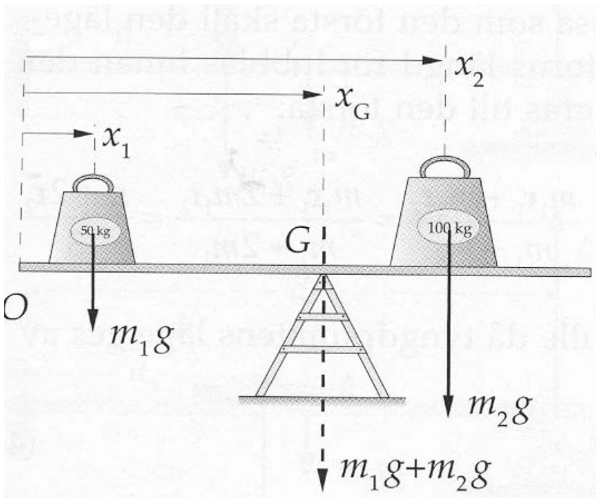


$$\mathbf{r}_G = \frac{\sum m_k \mathbf{r}_{gk}}{\sum m_k}$$

$$\mathbf{r}_G = \frac{\int \mathbf{r}_g \mathrm{d}m}{\int \mathrm{d}m}$$

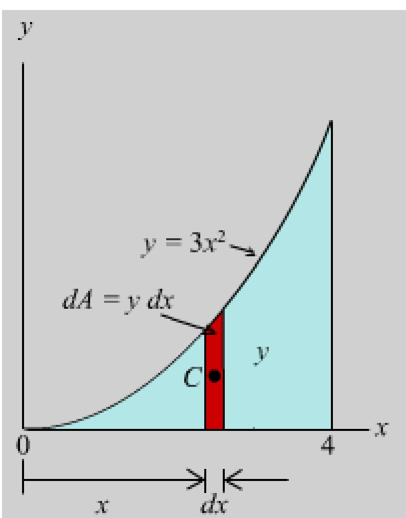
Masscentrum

Exempel



$$\circlearrowright O: \quad m_1 g \cdot x_1 + m_2 g \cdot x_2 = (m_1 + m_2) g \cdot x_G$$

$$\Rightarrow \quad x_G = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



$$\bar{y} = \frac{\int y_c \, dA}{\int dA} = \frac{\int \frac{y}{2} y dx}{\int y dx}$$

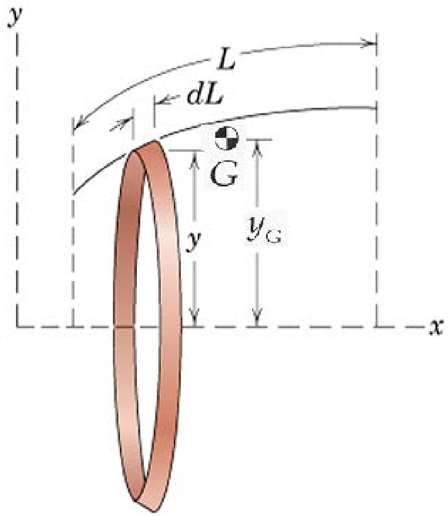
$$= \frac{\frac{1}{2}\int_0^4 (3x^2)^2 dx}{\int_0^4 3x^2 dx} = \frac{\frac{1}{2}(9)\frac{x^5}{5}\Big|_0^4}{3\,\frac{x^3}{3}\Big|_0^4}$$

$$= \frac{\frac{9}{10}(4)^5}{(4)^3} = \frac{72}{5}$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{\int xy\,dx}{\int y\,dx}$$

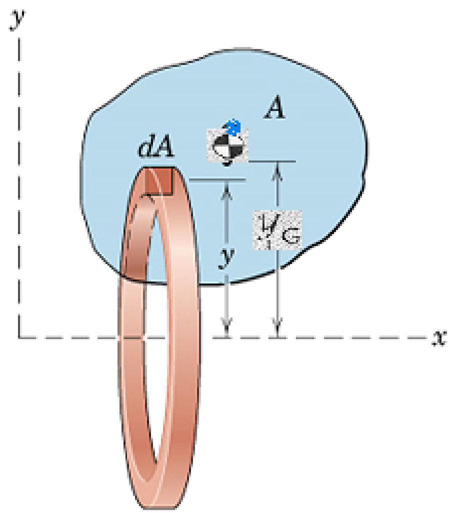
$$= \frac{\int_0^4 x(3x^2)dx}{\int_0^4 3x^2 dx} = \frac{3\frac{x^4}{4}\Big|_0^4}{3\frac{x^3}{3}\Big|_0^4}$$

$$= \frac{3}{4}\frac{4^4}{4^3} = 3$$



Papus första regel - rotationsarea

$$A = 2\pi y_G L$$

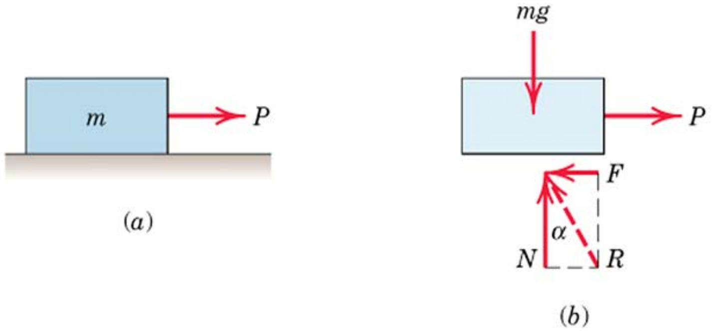


Papus andra regel - rotationsvolym

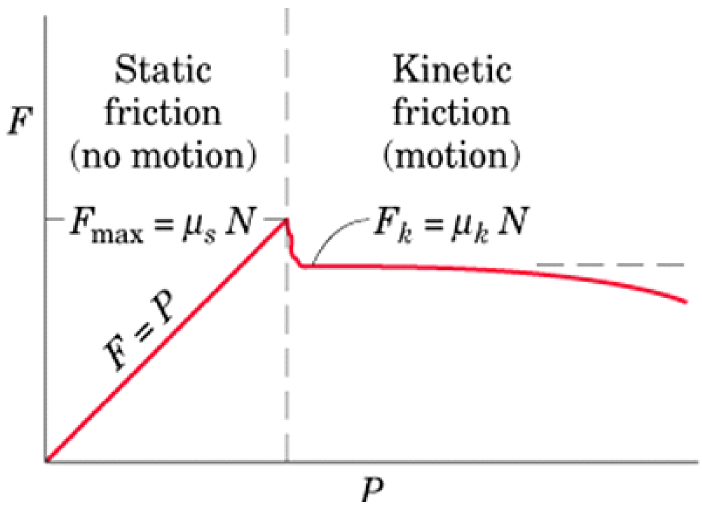
$$V = 2\pi y_G A$$

Friktion

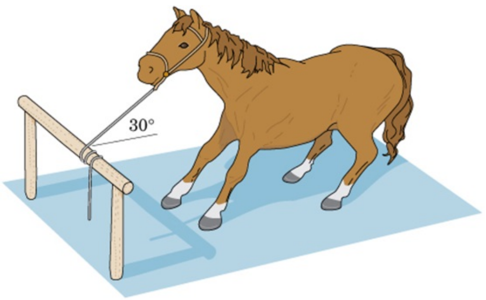
Torr (Coulomb)friktion

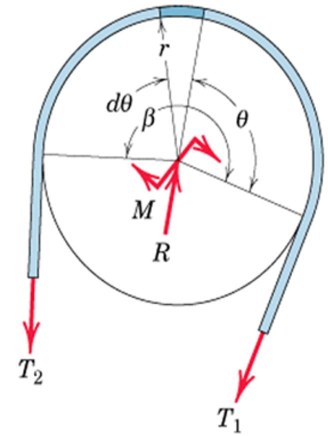


$$F \le \mu_s N$$



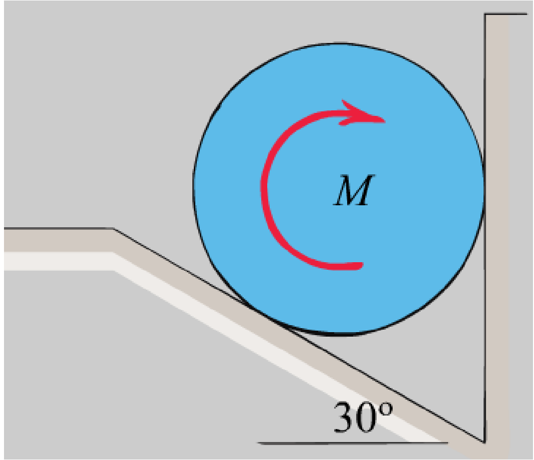
Remfriktion



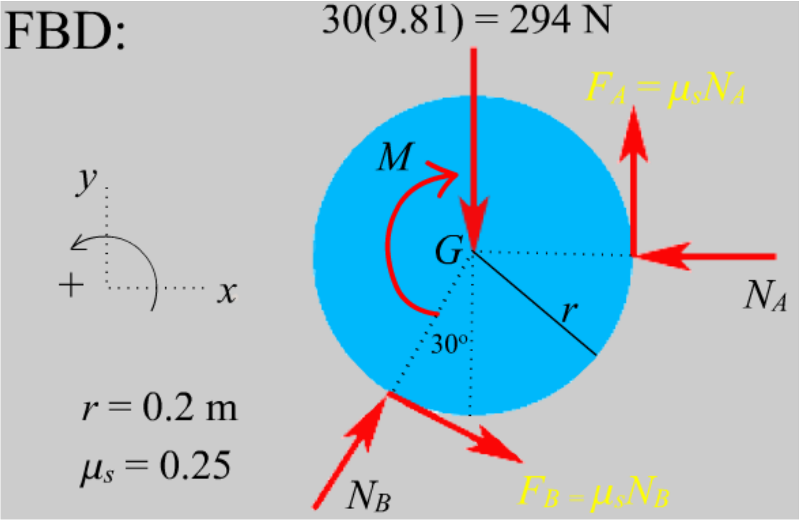


$$T_2 = e^{\mu\beta} \; T_1$$

Exempel



Frilägg



$$\Sigma F_x = 0: \quad N_B \sin 30^\circ + 0.25\, N_B \cos 30^\circ - N_A = 0$$

$$\Sigma F_y = 0: \quad N_B \cos 30^\circ - 0.25\, N_B \sin 30^\circ + 0.25\, N_A - 294 = 0$$

$$\Sigma M_G = 0: \quad -M + 0.25\, N_B (0.2) + 0.25\, N_A (0.2) = 0$$

$$N_A = 191.6 \text{ N}$$

$$N_B = 268 \text{ N}$$

$$M = 22.9 \text{ N·m}$$

Partikelkinematik

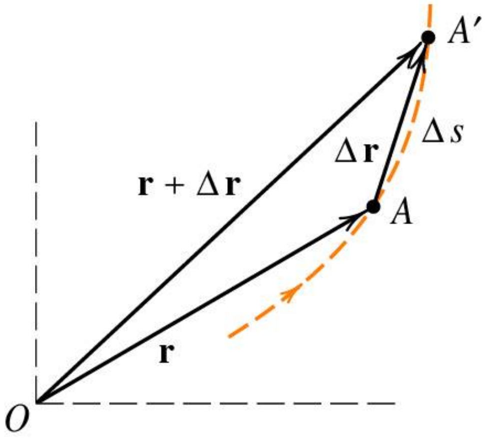
- Läge (position) $\mathbf{r} = \mathbf{r}(t)$

- Hastighet $\mathbf{v}(t) = \dot{\mathbf{r}}(t) = \frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}}$

- Medelhastighet $\mathbf{v}_{medel} = \frac{\Delta\mathbf{r}}{\Delta \mathrm{t}} = \frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t}$

- Fart $v = |\mathbf{v}| = \left|\frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}}\right|$

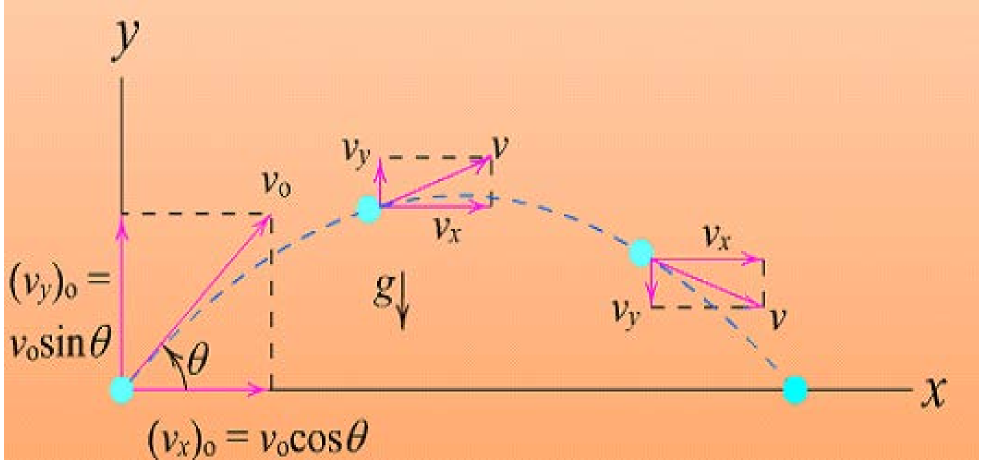
- Acceleration $\mathbf{a}(t) = \ddot{\mathbf{r}}(t) = \frac{\mathrm{d}\mathbf{v}}{\mathrm{dt}} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{ds}}\frac{\mathrm{ds}}{\mathrm{dt}} = v\frac{\mathrm{d}\mathbf{v}}{\mathrm{ds}}$



Kartesiska koordinater $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$

$$\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z \qquad \mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{e}_x + \dot{y}\mathbf{e}_y + \dot{z}\mathbf{e}_z \qquad \mathbf{a} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{e}_x + \ddot{y}\mathbf{e}_y + \ddot{z}\mathbf{e}_z$$

Kastparabel



$$\ddot{\mathbf{x}} = 0 \qquad \ddot{\mathbf{y}} = -g$$

$$\dot{\mathbf{x}} = v_0 \cos\beta \qquad \dot{\mathbf{y}} = -gt + v_0 \sin\beta$$

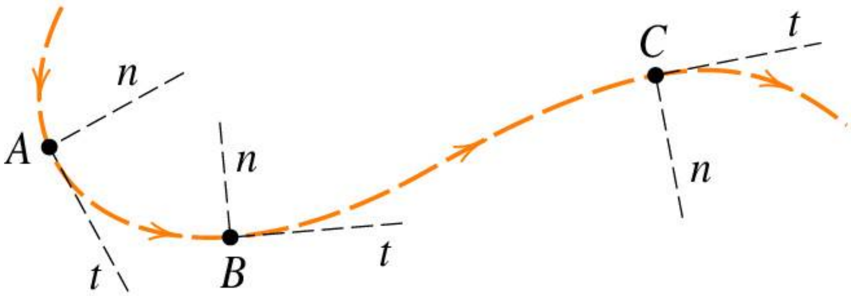
$$\mathbf{x} = v_0 \cos\beta\ t \qquad \mathbf{y} = -\frac{1}{2}gt^2 + v_0 \sin\beta\ t$$

Bankurvan: eliminera t $\quad \Rightarrow y = y(x) = -\frac{1}{2}g\left(\frac{x}{v_0 \cos\beta}\right)^2 + \left(\frac{v_0 \sin\beta}{v_0 \cos\beta}\right)x$

Kastvidd $\quad y = 0 \Rightarrow x = \frac{v_0^2}{g}\sin 2\beta$

Stighöjd $\quad \dot{y} = 0 \Rightarrow t = \frac{v_0}{g}\sin\beta \quad \Rightarrow y = \frac{v_0^2}{2g}\sin^2\beta$

Naturliga koordinater $(\mathbf{e}_t, \mathbf{e}_n, \mathbf{e}_b)$



Normal riktningen pekar mot Krökningskurvans centrum

Tangetial riktningen är tangenten till kurvans i rörelseriktningen

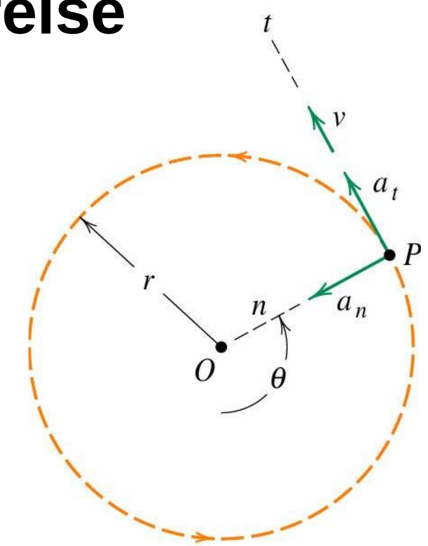
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{s}\mathbf{e}_t \qquad \mathbf{a} = \ddot{\mathbf{r}} = \ddot{s}\mathbf{e}_t + \frac{\dot{s}^2}{\rho}\mathbf{e}_n$$

cirkelrörelse

$$s = r\theta$$

$$\dot{s} = r\dot{\theta}$$

$$\ddot{s} = r\ddot{\theta}$$

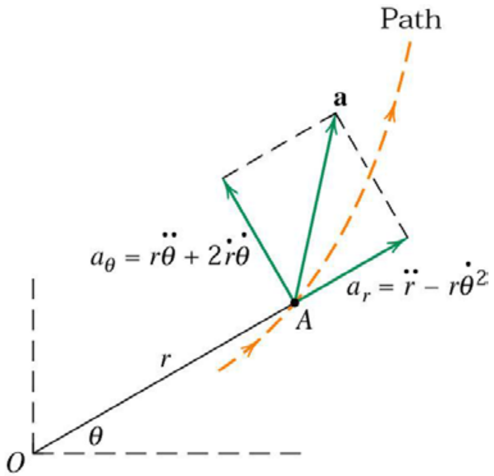


$$\mathbf{v} = \dot{s}\mathbf{e}_t = r\dot{\theta}\mathbf{e}_t$$

$$\mathbf{a} = \ddot{s}\mathbf{e}_t + \frac{\dot{s}^2}{\rho}\mathbf{e}_n = r\ddot{\theta}\mathbf{e}_t + \frac{(r\dot{\theta})^2}{r}\mathbf{e}_n$$

Cylinderkoordinater

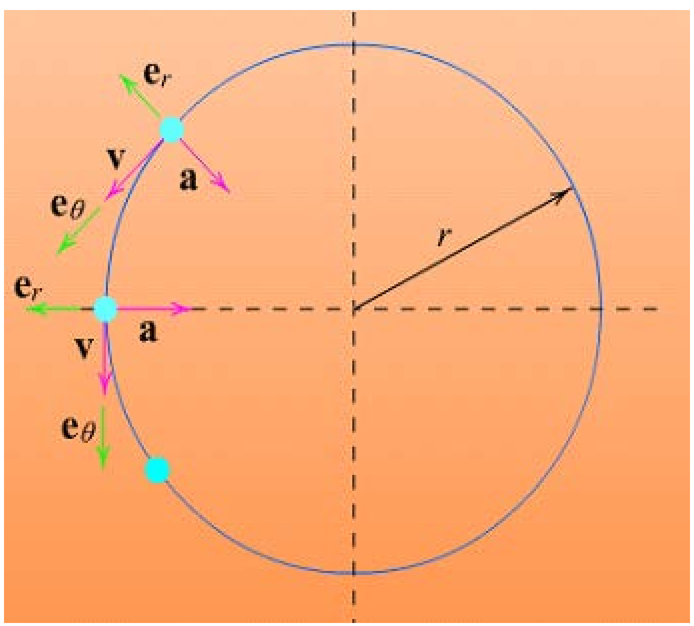
$$(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z)$$



$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{e}_z$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{e}_z$$

Cirkelrörelse med konstant fart

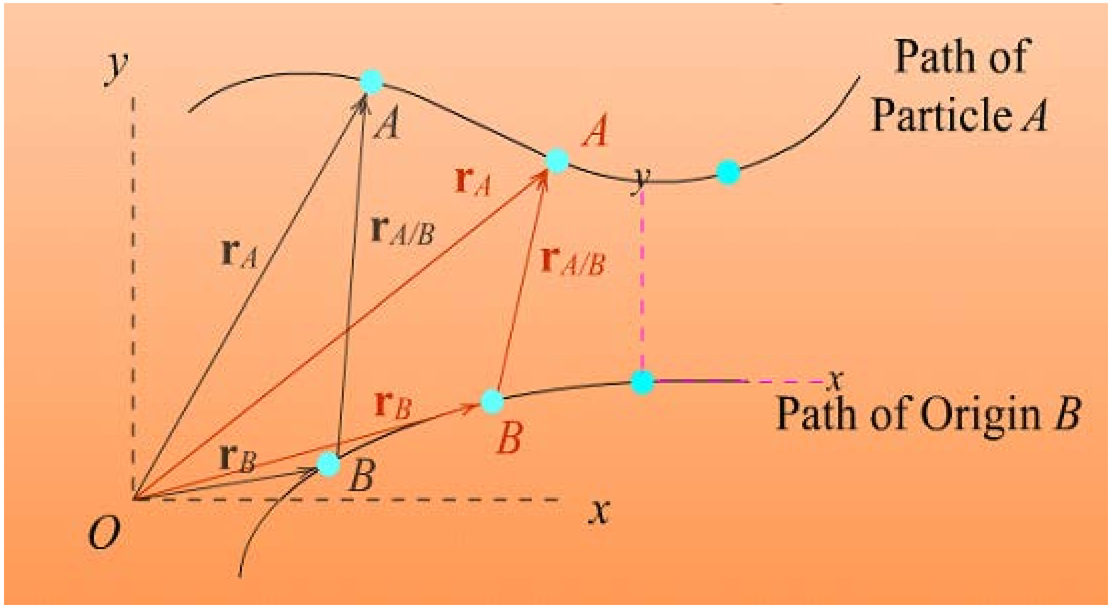


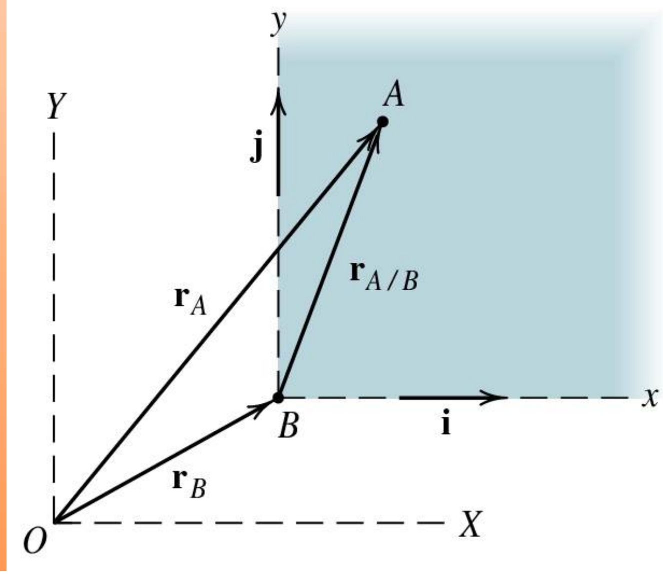
$$\mathbf{v} = v\mathbf{e}_\theta$$

$$\mathbf{a} = (\overset{0}{\cancel{\ddot{r}}} - r\dot{\theta}^2)\mathbf{e}_r + (\overset{0}{\cancel{r\ddot{\theta}}} + \overset{0}{\cancel{2\dot{r}\dot{\theta}}})\mathbf{e}_\theta$$

$$\mathbf{a} = -\frac{v^2}{r}\mathbf{e}_r$$

Relativ rörelse





$$\mathbf{r}_A = \mathbf{r}_A + \mathbf{r}_{A/B}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \qquad \mathbf{v}_{A/B} = \mathbf{v}_{REL}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} \qquad \mathbf{a}_{A/B} = \mathbf{a}_{REL}$$

Kraftekvationen

$$\mathbf{F} = m\mathbf{a}$$

Kartesiska koordinater

$$F_x = m\ddot{x}$$
$$F_y = m\ddot{y}$$
$$F_z = m\ddot{z}$$

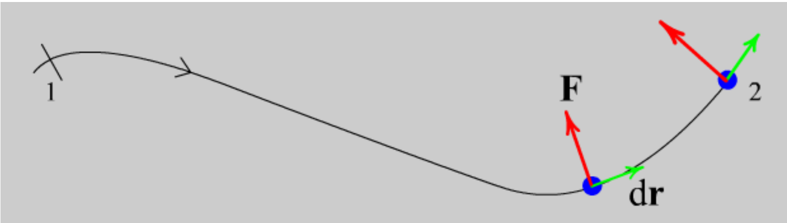
Naturliga koordinater

$$F_n = ma_n = m\frac{v^2}{\rho}$$
$$F_t = ma_t = mr\ddot{\theta}$$
$$F_b = 0$$

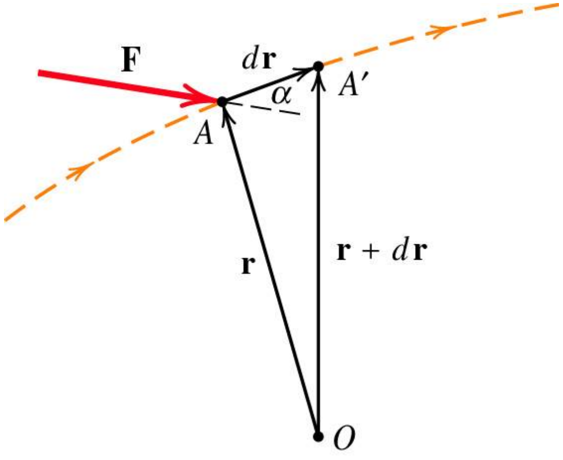
Cylinder koordinater

$$F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$
$$F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$
$$F_z = m\ddot{z}$$

Arbete och Energilagar



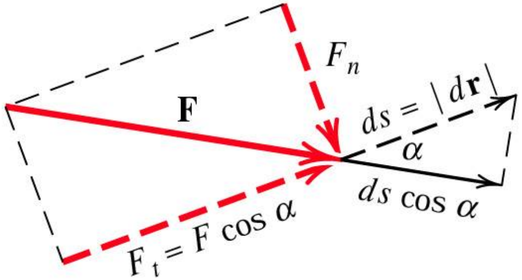
$$dU = F \cdot dr$$



(a)

$$F_t = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{ds}{dt}$$

$$\int_{s1}^{s2} F_t ds = \int_{s1}^{s2} mvdv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

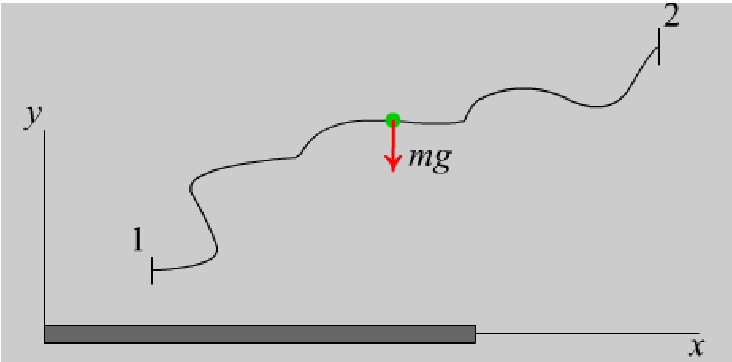


(b)

T - kinetisk energi

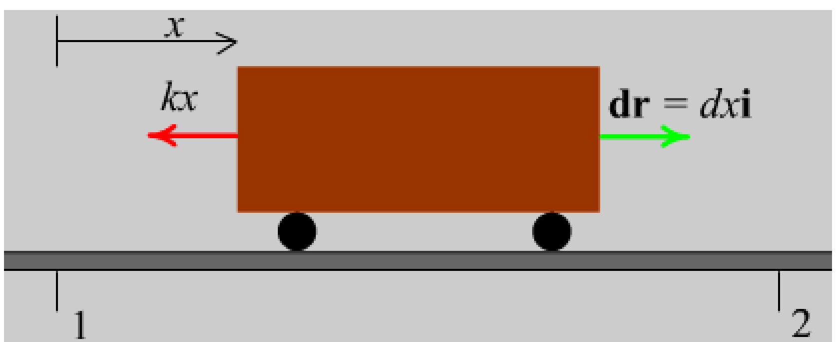
$$U_{1-2} = T_2 - T_1$$

Tyngdkraftens arbete



$$U = \int_{1}^{2} (F_x dx + F_y dy) = \int_{y1}^{y2} -mgdy = mg(y_1 - y_2)$$

Fjäderkraftens arbete



$$U = \int_{x1}^{x2} F_x dx = \int_{x1}^{x2} -kxdx = \frac{1}{2} k(x_1^2 - x_2^2)$$

Konservativa krafter - arbetet är oberoende av bankurvan och motsvaras av en potentialfunktion V

$$V = -\int F dr$$

Tyngdkraftens potentialfunktion

$$V = -\int_0^y -mgdy = mgy$$

Fjäderkraftens potentialfunktion

$$V = -\int_0^x -kxdx = \frac{1}{2}kx^2$$

Mekaniska energilagen

$$T + V = T_0 + V_0$$

Effekt

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \dot{\mathbf{r}} = \mathbf{F} \cdot \mathbf{v}$$

Enhet: 1Watt =1Nm/s 1hp=745.6 Watt

Impulsekvationen

Impuls-kraftens verkan under en viss tid

Newton andra lag ger:

$$\mathbf{F} = m\mathbf{a} = m \cdot \frac{d\mathbf{v}}{dt} = m \cdot \dot{\mathbf{v}} = \frac{d}{dt}(m\mathbf{v}) = \frac{d\mathbf{p}}{dt} = \dot{\mathbf{p}}$$

$$\int_{t_1}^{t_2} \mathbf{F} dt = \int_{\mathbf{p}_0}^{\mathbf{p}} d\mathbf{p} = \mathbf{p}(t) - \mathbf{p}(t_0)$$

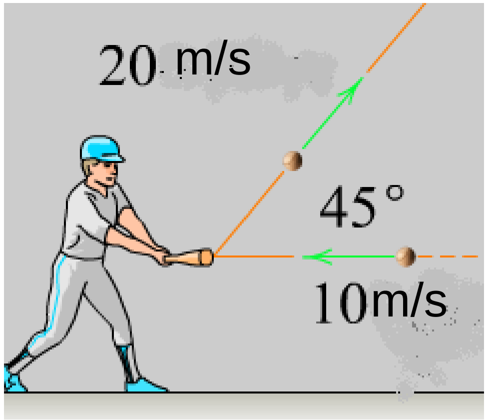
eller

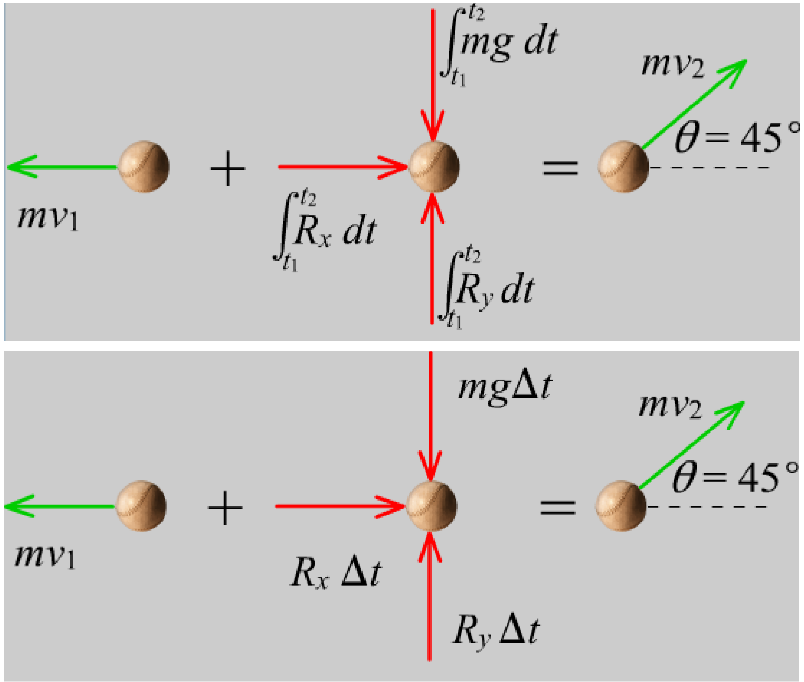
$$\mathbf{p}(t_0) + \int_{t_1}^{t_2} \mathbf{F} dt = \mathbf{p}(t)$$

Om inga yttre krafter verkar på systemet,
dvs **F**=0 bevaras systemets rörelsemängd

$$\mathbf{p}(t) = \mathbf{p}(t_0)$$

Exempel:





Bestäm kraften R som påverkar
Bollen om slaget varar i 0.05 s

$$\Delta t = 0.05\ s$$

$$m = 0.5\ kg$$

Impulsekvationen i x- och y-led

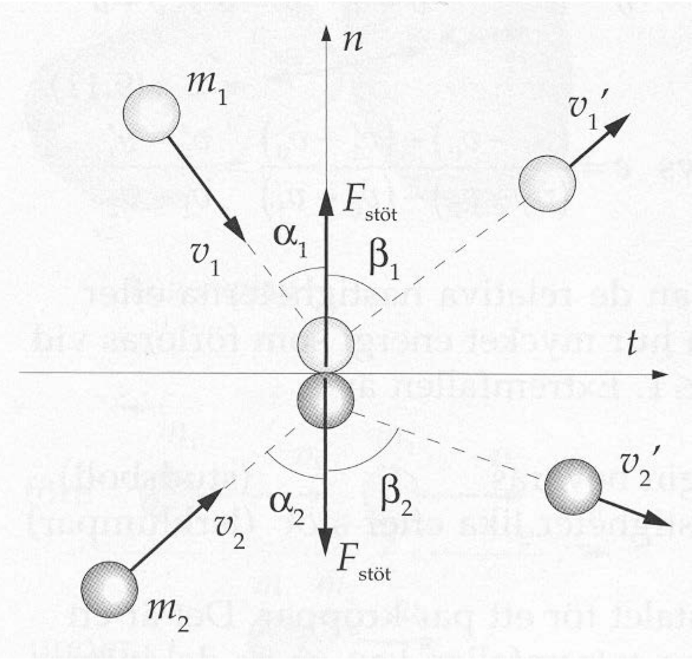
$$mv_{1x} + \int_{t_1}^{t_2} \sum F_x dt = mv_{2x} : -mv_1 + R_x\Delta t = mv_2\cos\theta$$

$$mv_{1y} + \int_{t_1}^{t_2} \sum F_y dt = mv_{2y} : 0 + R_y\Delta t - mg\Delta t = mv_2\sin\theta$$

Sned central stöt

Rörelsemängden för hela systemet bevaras i n-led

1. $$-m_1 v_1 \cos\alpha_1 + m_2 v_2 \cos\alpha_2 = m_1 v_1' \cos\beta_1 - m_2 v_2' \cos\beta_2$$



Rörelsemängden för vardera kroppen bevaras i t-led

2. $$m_1 v_1 \sin\alpha_1 = m_1 v_1' \sin\beta_1$$

3. $$m_2 v_2 \sin\alpha_2 = m_2 v_2' \sin\beta_2$$

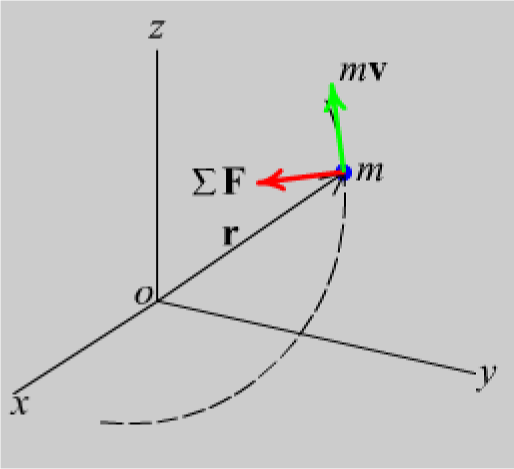
Studstalet e är definierad i n-led

4. $$e = \frac{v_1' \cos\beta_1 - \left(-v_2' \cos\beta_2\right)}{v_2 \cos\alpha_2 - \left(-v_1 \cos\alpha_1\right)}$$

Rörelsemämngdsmoment

$$\mathbf{H}_0 = \mathbf{r} \times m\mathbf{v}$$

Differentiera med avseende på tiden t:



$$\dot{\mathbf{H}}_0 = \underbrace{\dot{\mathbf{r}} \times m\mathbf{v}}_{parallela\ vektorer = 0} + \mathbf{r} \times m\dot{\mathbf{v}} = \mathbf{r} \times m\dot{\mathbf{v}}$$

Insättning av Newtons andra lag ger:

$$\dot{\mathbf{H}}_0 = \mathbf{r} \times m\dot{\mathbf{v}} = \mathbf{r} \times m\mathbf{a} = \mathbf{r} \times \mathbf{F} = \mathbf{M}_0$$

Momentekvationen

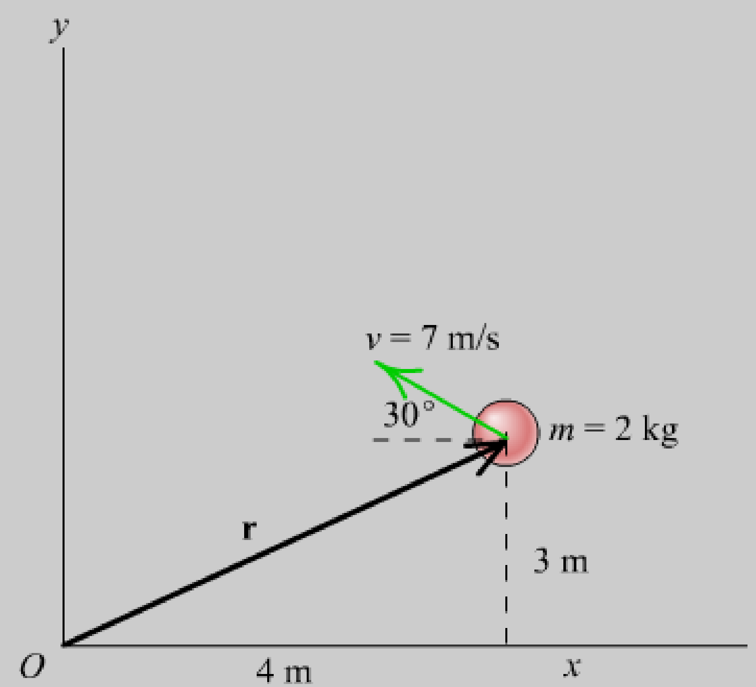
$$\mathbf{M}_0 = \dot{\mathbf{H}}_0 = \frac{d\mathbf{H}_0}{dt}$$

Integrering ger impulsmomentekvationen:

$$\int_{t_0}^{t} \mathbf{M}_0 dt = \mathbf{H}_0(t) - \mathbf{H}_0(t_0)$$

Rörelsemängdsmomentet bevaras om
Inget impulsmoment verkar på systemet:

$$\mathbf{H}_0(t) = \mathbf{H}_0(t_0)$$



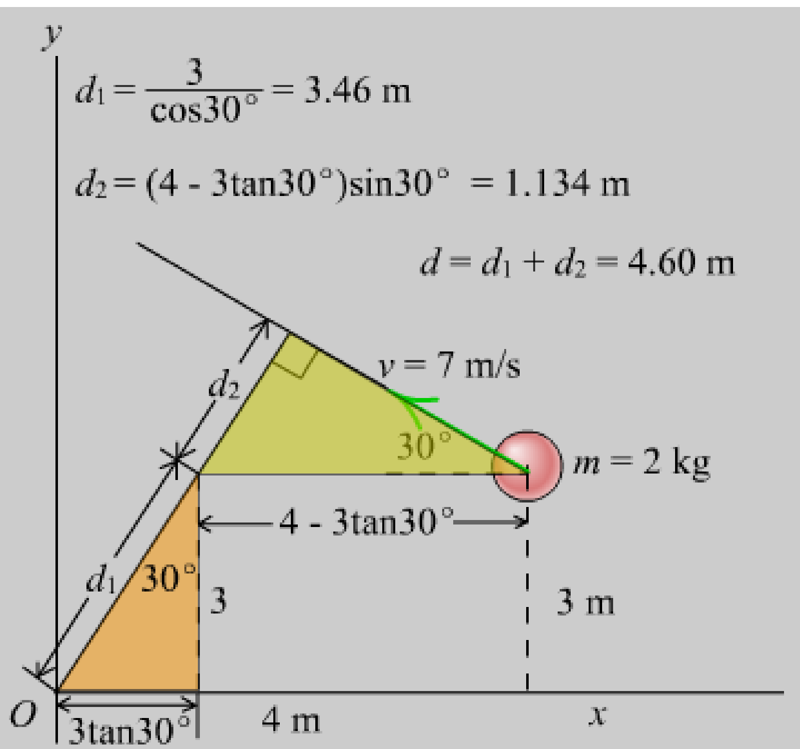
Exempel: Bestäm bollens rörelsemängdsmoment kring O på fem olika sätt:

1. $$\mathbf{H}_0 = \mathbf{r} \times m\mathbf{v}$$

$$\mathbf{r} = 4\mathbf{e}_x + 3\mathbf{e}_y$$

$$\mathbf{v} = (-7\cos 30\mathbf{e}_x + 7\sin 30\mathbf{e}_y)$$

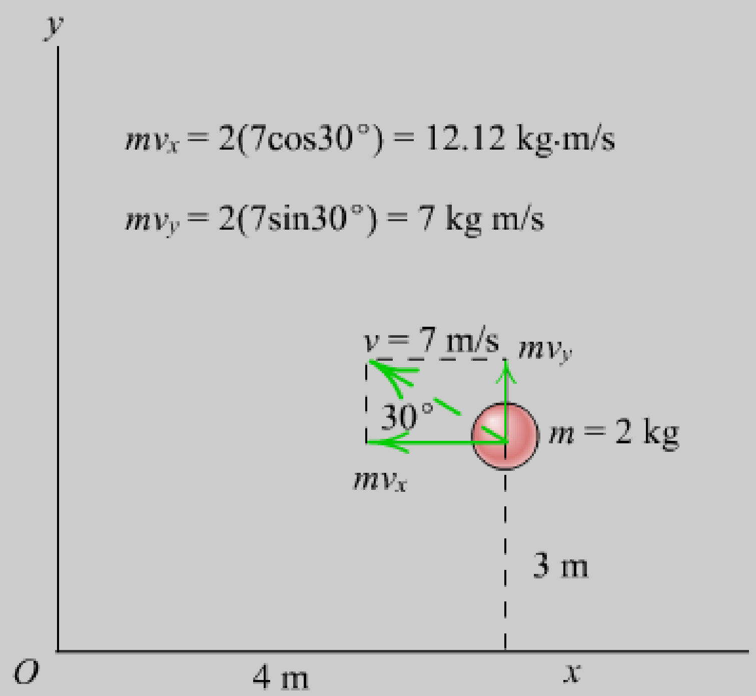
$$\mathbf{H}_0 = m(\mathbf{r} \times \mathbf{v}) = 64.4\ \mathbf{e}_z\ \text{kg m/s}^2$$



2. $$H_0 = mvd$$

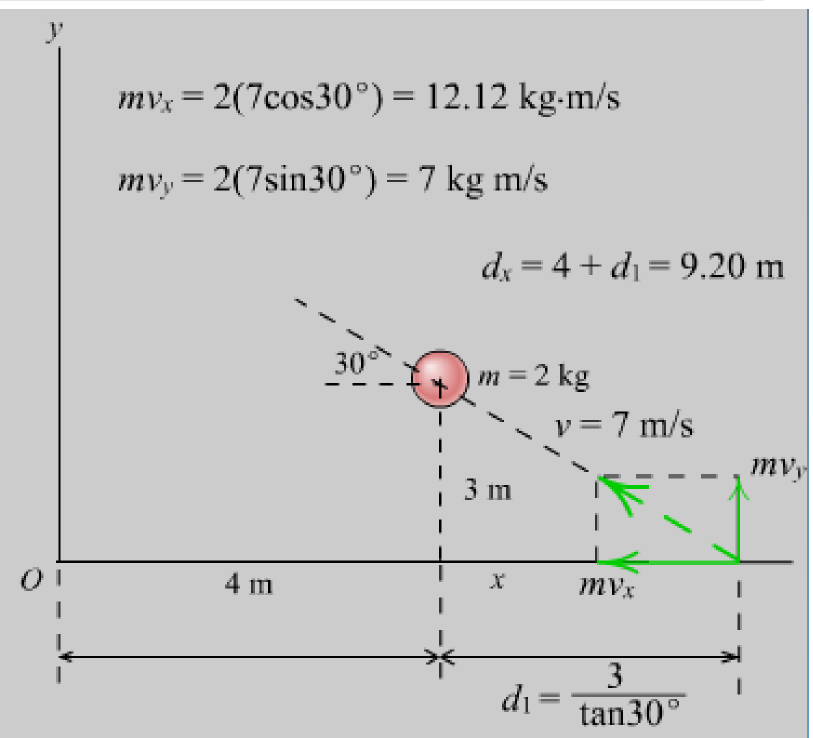
$$H_0 = mvd = 2 \cdot 7 \cdot 4.6 = 64.4\ \text{kg m/s}^2$$

moturs



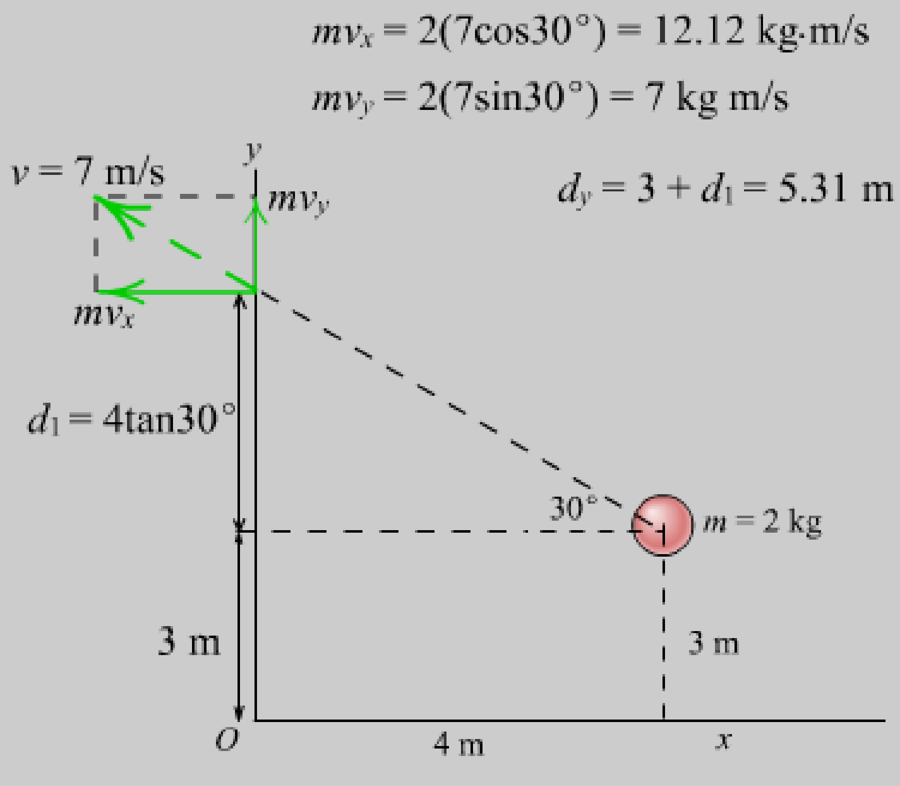
3. $$H_0 = mv_x \cdot 3 + mv_y \cdot 4 = 12.12 \cdot 3 + 7 \cdot 4$$

$$H_0 = 64.4 \text{ kg m/s}^2$$ moturs



4. $$H_0 = mv_y d_x = 7 \cdot 9.2$$

$$H_0 = 64.4 \text{ kg m/s}^2$$ moturs



5. $$H_0 = mv_x d_y = 12.12 \cdot 5.31$$

$$H_0 = 64.4 \text{ kg m/s}^2$$ moturs