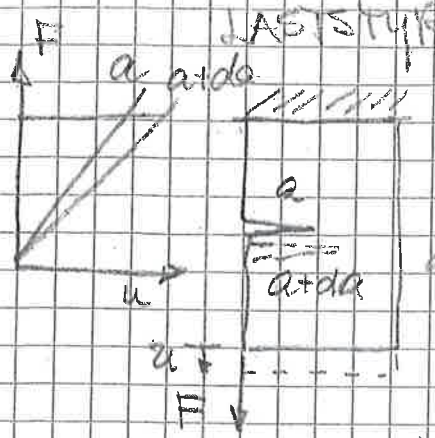
(1)

$W_p = \Phi - W_F$ = el energi - ytre krafters arb

elastic energy - work of external forces

LASTSTYRT



$A = Ba$

$dA = B\,da$

FÖRSKJUTNINGSSTYRT

$k = \frac{F}{u}$

$$\begin{cases} \Phi\big|_F = \int_0^u F\,du = \int_0^u ku\,du = \frac{Fu}{2} \\ W_F = Fu \end{cases}$$

stiffness

$W_p = -\frac{1}{2}Fu$ (INSTABILT)

$\mathcal{G} = -\frac{dW_p}{dA}$

$\mathcal{G}\big|_F = -\frac{1}{B}\frac{d}{da}\left(-\frac{1}{2}Fu\right)\Big|_F = \frac{1}{2B}\frac{d}{da}\left(F\cdot\frac{F}{k}\right)\Big|_F = \frac{F^2}{2B}\frac{dC}{da}\Big|_F = \frac{F^2}{2}\frac{dC}{dA}\Big|_F$; $C = \frac{1}{k}$

$F = ku$

$$\begin{cases} \Phi\big|_u = \int_0^F u\,dF = \int_0^F uk\,du = \frac{Fu}{2} \\ W_F = 0 \end{cases}$$

$W_p = \frac{1}{2}Fu$ (STABILT)

$\mathcal{G} = -\frac{dW_p}{da}$

$\mathcal{G}\big|_u = -\frac{1}{B}\frac{d}{da}\left(\frac{1}{2}Fu\right)\Big|_u = -\frac{1}{2B}\frac{d}{da}(ku\cdot u)\Big|_u = -\frac{u^2}{2B}\frac{d}{da}\left(\frac{1}{C}\right)\Big|_u = -\frac{u^2}{2B}\cdot\frac{-1}{C^2}\frac{dC}{da}\Big|_u = \frac{u^2}{2BC^2}\frac{dC}{da}\Big|_u = \frac{F^2}{2B}\frac{dC}{da}\Big|_u = \frac{F^2}{2}\frac{dC}{dA}\Big|_u$

$\mathcal{G}$ = Energy release rate

compliance

C = kompliansen = $\frac{1}{\text{styrheten}} = \frac{1}{k}$

stiffness

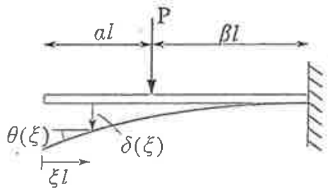
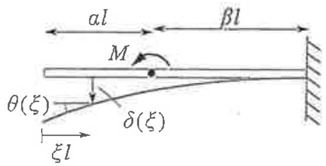
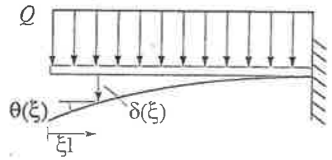
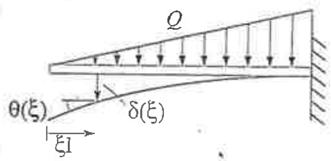
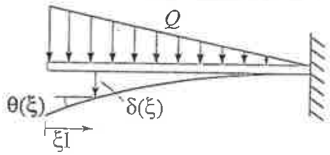
If you know G_0, you know $\mathcal{G}$

Fracture

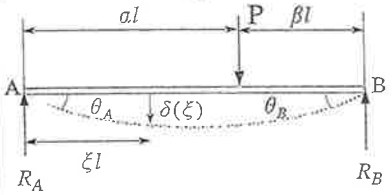
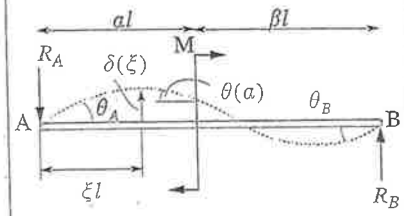
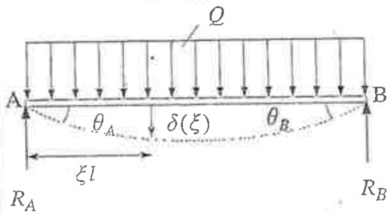
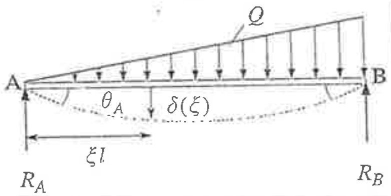
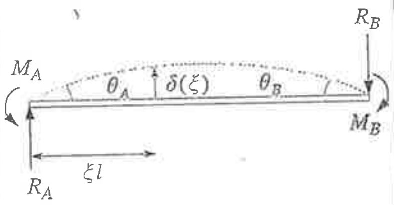
VET MAN G_0 SÅ VET MAN $\mathcal{G}$. BROTT: $\mathcal{G} = \mathcal{G}_c$

$\frac{d\mathcal{G}}{dA} > 0$ INSTABILT

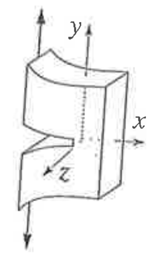
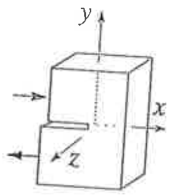
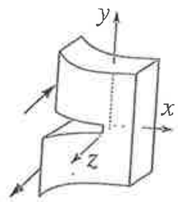
$\frac{d\mathcal{G}}{dA} < 0$ STABILT

Belastningsfall, $\alpha + \beta = 1$	θ = vinkeländring, $\delta(\xi = x/l)$ = förskjutning
	$\theta(\xi < \alpha) = \frac{Pl^2}{2EI}\beta^2$ $\delta(\xi < \alpha) = \frac{Pl^3}{6EI}(-\beta^3 + 3\beta^2(1-\xi))$ $\theta(\xi > \alpha) = \frac{Pl^2}{2EI}(\beta^2 - (\xi-\alpha)^2)$ $\delta(\xi > \alpha) = \frac{Pl^3}{6EI}((\xi-\alpha)^3 - 3\beta^2(\xi-\alpha) + 3\beta^2)$ $\theta(\alpha) = \frac{Pl^2}{2EI}\beta^2 \quad \delta(\alpha) = \frac{Pl^3}{3EI}\beta^3$
	$\theta(\xi < \alpha) = \frac{Ml}{EI}\beta$ $\delta(\xi < \alpha) = \frac{Ml^2}{EI}\beta(1 - \xi - \beta/2)$ $\theta(\xi > \alpha) = \frac{Ml}{EI}(1-\xi)$ $\delta(\xi > \alpha) = \frac{Ml^2}{2EI}((\xi-\alpha)^2 - 2\beta(\xi-\alpha) + \beta^2)$ $\theta(\alpha) = \frac{Ml}{EI}\beta \quad \delta(\alpha) = \frac{Ml^2}{2EI}\beta^2$
	$\theta(\xi) = \frac{Ql^2}{6EI}(1-\xi^3)$ $\delta(\xi) = \frac{Ql^3}{24EI}(\xi^4 - 4\xi + 3)$
	$\theta(\xi) = \frac{Ql^2}{12EI}(1-\xi^4)$ $\delta(\xi) = \frac{Ql^3}{60EI}(\xi^5 - 5\xi + 4)$
	$\theta(\xi) = \frac{Ql^2}{12EI}(\xi^4 - 4\xi^3 + 3)$ $\delta(\xi) = \frac{Ql^3}{60EI}(-\xi^5 + 5\xi^4 - 15\xi + 11)$

Tabell 32.2 Elementarfall för fritt upplagd balk

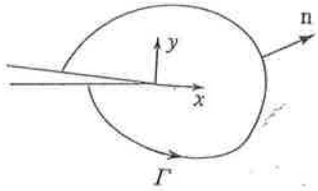
Belastningsfall, $(\alpha + \beta = 1)$	θ = vinkeländring, $\delta(\xi = x/l)$ = förskjutning
1. 	$R_A = \beta P \;;\; R_B = \alpha P$ $\theta_A = \frac{Pl^2}{6EI}\alpha\beta(1+\beta) \;;\; \theta_B = \frac{Pl^2}{6EI}\alpha\beta(1+\alpha)$ $\delta(\xi) = \frac{Pl^3}{6EI}\beta[(1-\beta^2)\xi - \xi^3]$ för $\xi \le \alpha$ $\delta(\alpha) = \frac{Pl^3}{3EI}\alpha^2\beta^2$
2. 	$R_A = R_B = M/l$ $\theta_A = \frac{Ml}{6EI}(1-3\beta^2) \;;\; \theta_B = \frac{Ml}{6EI}(1-3\alpha^2)$ $\delta(\xi) = \frac{Ml^2}{6EI}((1-3\beta^2)\xi - \xi^3)$ för $\xi \le \alpha$ $\delta(\alpha) = \frac{Ml^2}{3EI}\alpha\beta(\alpha-\beta) \;;\; \theta(\alpha) = \frac{Ml}{3EI}(1-3\alpha$
3. 	$R_A = R_B = Q/2$ $\theta_A = \theta_B = \frac{Ql^2}{24EI}$ $\delta(\xi) = \frac{Ql^3}{24EI}(\xi - 2\xi^3 + \xi^4)$ $\delta\left(\frac{1}{2}\right) = \frac{5}{384}\cdot\frac{Ql^3}{EI}$
4. 	$R_A = Q/3 \;;\; R_B = 2Q/3$ $\theta_A = \frac{7Ql^2}{180EI} \;;\; \theta_B = \frac{8Ql^2}{180EI}$ $\delta(\xi) = \frac{Ql^3}{180EI}(7\xi - 10\xi^3 + 3\xi^5)$
5. 	$R_A = R_B = (M_A - M_b)/l$ $\theta_A = \frac{M_A l}{3EI} + \frac{M_B l}{6EI} \;;\; \theta_B = \frac{M_A l}{6EI} + \frac{M_B l}{3EI}$ $\delta(\xi) = \frac{l^2}{6EI}[M_A(2\xi - 3\xi^2 + \xi^3) + M_B(\xi - \xi$ $M_A = M_B = M: \theta_A = \theta_B = \frac{Ml}{2EI};$ $\delta(\xi) = \frac{Ml^2}{2EI}(\xi - \xi^2)$

9.3.1
9.3.2

Modus	*I*	*II*	*III*
Förskjut-nings-diskontinuitet	u_y	u_x	u_z
	 *Vidgande i xy-planet*	 *Glidande i xy-planet*	 *Glidande tvärs xy-planet*

Figur 23.1 Definition av brottmekaniska modus

23.3 J-INTEGRALEN



Figur 23.2 Definitioner av storheter för *J*-integralen

För två-dimensionella sprickproblem definieras en integral J tagen längs en väg Γ runt en sprickspets (figur 23.2) enligt

$$J = \int_{\Gamma} \left(A' dy - \sigma_{ij} n_j \frac{\partial u_i}{\partial x} ds \right) \qquad (23.1)$$

där $A' = \int \sigma_{ij} d\varepsilon_{ij}$ är deformationsarbetet per volymsenhet.

För homogena, elastiska material där $A' = W'$ är en töjningsenergifunktion med egenskapen

$$\sigma_{ij} = \frac{\partial W'}{\partial \varepsilon_{ij}} \tag{23.2}$$

är J oberoende av vägen Γ.

För sådana material gäller med potentiella energin U enligt (7.11)

$$J = -\frac{1}{nt}\frac{\partial U}{\partial a} \tag{23.3}$$

där $n=1$ för kantspricka och $n=2$ för inre spricka med symmetriska förhållanden, t är kroppens tjocklek och a är spricklängden.

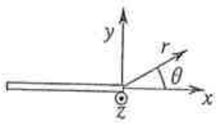
23.4 LINJÄRT ELASTISK BROTTMEKANISK TEORI

I den *linjärt elastiska brottmekaniken* antas strukturen kunna beskrivas med enbart linjär isotrop elastisk teori. Tillståndet vid en sprickspets beskrivs då entydigt av *spänningsintensitetsfaktorerna* K_I, K_{II} och K_{III} definierade enligt (23.4)-(23.6).

$$K_I = \lim_{x \to +0} \sigma_y(x, 0)\sqrt{2\pi x} \tag{23.4}$$

$$K_{II} = \lim_{x \to +0} \tau_{xy}(x, 0)\sqrt{2\pi x} \tag{23.5}$$

$$K_{III} = \lim_{x \to +0} \tau_{yz}(x, 0)\sqrt{2\pi x} \tag{23.6}$$



Figur 23.3 Koordinater vid sprickspets

23.4.1 Asymptotiska lösningar vid sprickspetsen

Spänningar och förskjutningar i sprickspetsens närhet kan skrivas enligt (23.7) - (23.12). Härvid är κ definierat enligt

$$\kappa = \begin{cases} 3-4\nu & \text{plan deformation} \\ (3-\nu)/(1+\nu) & \text{plan spänning} \end{cases}$$

Värdet hos spänningsintensitetsfaktorerna K_I, K_{II} och K_{III} beror av det aktuella randvärdesproblemet enligt avsnitt 23.4.3.

Modus I

$$\begin{cases} \sigma_x = \dfrac{K_\mathrm{I}}{\sqrt{2\pi r}}\left(\cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)\right) + \sigma_{xx0} + O(\sqrt{r}) \\ \sigma_y = \dfrac{K_\mathrm{I}}{\sqrt{2\pi r}}\left(\cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)\right) + O(\sqrt{r}) \\ \tau_{xy} = \dfrac{K_\mathrm{I}}{\sqrt{2\pi r}}\left(\cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}\right) + O(\sqrt{r}) \\ \tau_{zx} = \tau_{yz} = 0 \\ \sigma_z = \begin{cases} \nu\sigma_x + \sigma_y & \text{plan deformation} \\ 0 & \text{plan spänning} \end{cases} \end{cases} \tag{23.7}$$

$$\begin{cases} u = \dfrac{(1+\nu)K_\mathrm{I}}{4\pi E}\sqrt{2\pi r}\left((2\kappa - 1)\cos\dfrac{\theta}{2} - \cos\dfrac{3\theta}{2}\right) + O(r) \\ v = \dfrac{(1+\nu)K_\mathrm{I}}{4\pi E}\sqrt{2\pi r}\left((2\kappa + 1)\sin\dfrac{\theta}{2} - \sin\dfrac{3\theta}{2}\right) + O(r) \\ w = \begin{cases} 0 & \text{plan deformation} \\ -\dfrac{\nu}{E}\displaystyle\int(\sigma_x + \sigma_y)dz & \text{plan spänning} \end{cases} \end{cases} \tag{23.8}$$

Modus II

$$\begin{cases} \sigma_x = -\dfrac{K_\mathrm{II}}{\sqrt{2\pi r}}\left(\sin\dfrac{\theta}{2}\left(2 + \cos\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}\right)\right) + O(\sqrt{r}) \\ \sigma_y = \dfrac{K_\mathrm{II}}{\sqrt{2\pi r}}\left(\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\cos\dfrac{3\theta}{2}\right) + O(\sqrt{r}) \\ \tau_{xy} = \dfrac{K_\mathrm{II}}{\sqrt{2\pi r}}\left(\cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right)\right) + O(\sqrt{r}) \\ \tau_{xz} = \tau_{yz} = 0 \\ \sigma_z = \begin{cases} \nu(\sigma_x + \sigma_y) & \text{plan deformation} \\ 0 & \text{plan spänning} \end{cases} \end{cases} \tag{23.9}$$

Modus II forts

$$\begin{cases} u = \dfrac{(1+\nu)K_{II}\sqrt{2\pi r}}{4\pi E}\left((2\kappa+3)\sin\dfrac{\theta}{2}+\sin\dfrac{3\theta}{2}\right)+O(r) \\ v = \dfrac{(1+\nu)K_{II}\sqrt{2\pi r}}{4\pi E}\left((2\kappa-3)\cos\dfrac{\theta}{2}+\cos\dfrac{3\theta}{2}\right)+O(r) \\ w = \begin{cases} 0 & \text{plan deformation} \\ -\dfrac{\nu}{E}\int(\sigma_x+\sigma_y)dz & \text{plan spänning} \end{cases} \end{cases} \qquad (23.10)$$

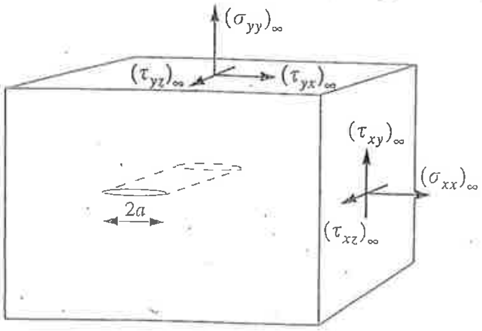
Modus III

$$\begin{cases} \tau_{xz} = -\dfrac{K_{III}}{\sqrt{2\pi r}}\sin\dfrac{\theta}{2}+O(\sqrt{r}) \\ \tau_{yz} = \dfrac{K_{III}}{\sqrt{2\pi r}}\cos\dfrac{\theta}{2}+O(\sqrt{r}) \\ \sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0 \end{cases} \qquad (23.11)$$

$$\begin{cases} w = \dfrac{2(1+\nu)K_{III}}{\pi E}\sqrt{2\pi r}\sin\dfrac{\theta}{2}+O(r^{3/2}) \\ u = v = 0 \end{cases} \qquad (23.12)$$

23.4.2 K_I, K_{II} och K_{III} för lång spricka i helrymd

Ett intressant resultat är spänningsintensitetsfaktorer för en spricka av längd $2a$ i en oändlig solid utsatt för ett homogent avläget spänningstillstånd $(\sigma_{ij})_\infty$



$$K_I = (\sigma_{yy})_\infty\sqrt{\pi a}$$

$$K_{II} = (\tau_{yx})_\infty\sqrt{\pi a}$$

$$K_{III} = (\tau_{yz})_\infty\sqrt{\pi a}$$

Figur 23.4 Spänningsintensitetsfaktorer för spricka i helrymd

W >> a W = 1 m B = 0.01 m $a_0 = 2$ mm $a_f = 5$ mm

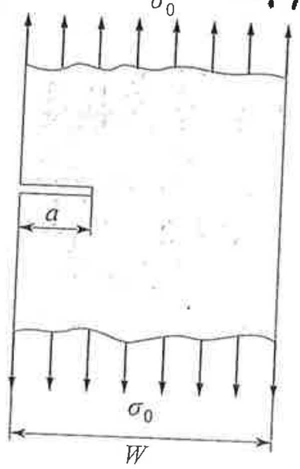
$K_{Ic} = 52$ MNm$^{3/2}$ $\sigma_y = 1200$ MPa m = 4 $C = 2{,}7\text{-}26\,10^{-14}$ MPa

Kantspricka i strimla utsatt för enaxlig dragning

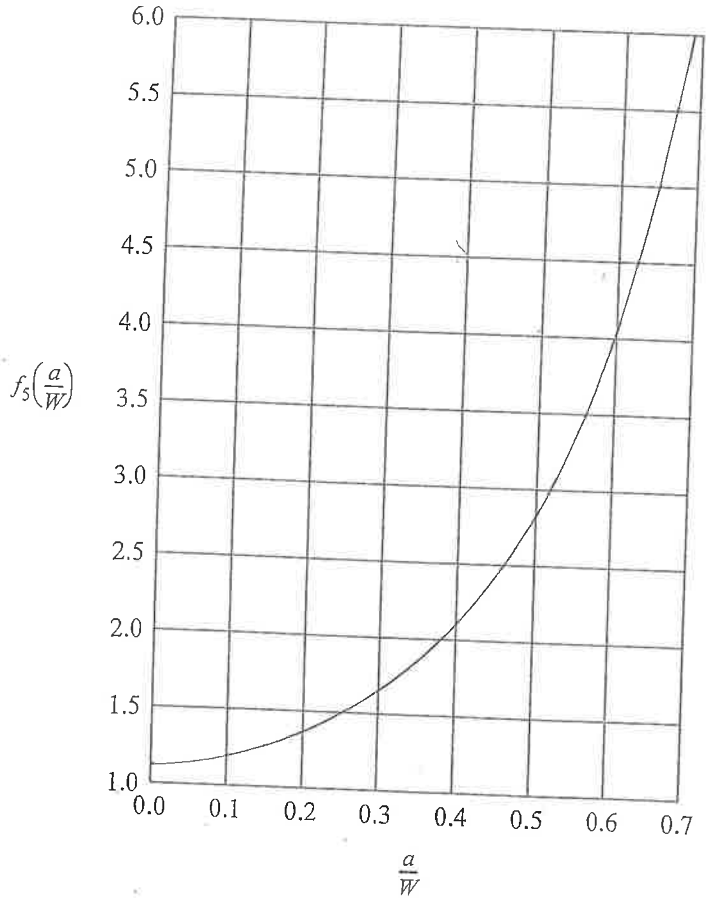
$$K_I = \sigma_0 \sqrt{\pi a}\; f_5\left(\frac{a}{W}\right)$$

$$f_5\left(\frac{a}{W}\right) = \left(\frac{2W}{\pi a}\tan\frac{\pi a}{2W}\right)^{1/2}\left(\cos\frac{\pi a}{2W}\right)^{-1}$$

$$\left(0{,}752 + 2{,}02\left(\frac{a}{W}\right) + 0{,}37\left(1 - \sin\frac{\pi a}{2W}\right)^3\right)$$



H. TADA, P.C. PARIS, G. R. IRWIN,
The Stress Analysis of Cracks Handbook,
Del Research Corp, 1973.



Geometrifunktion för fall 5.

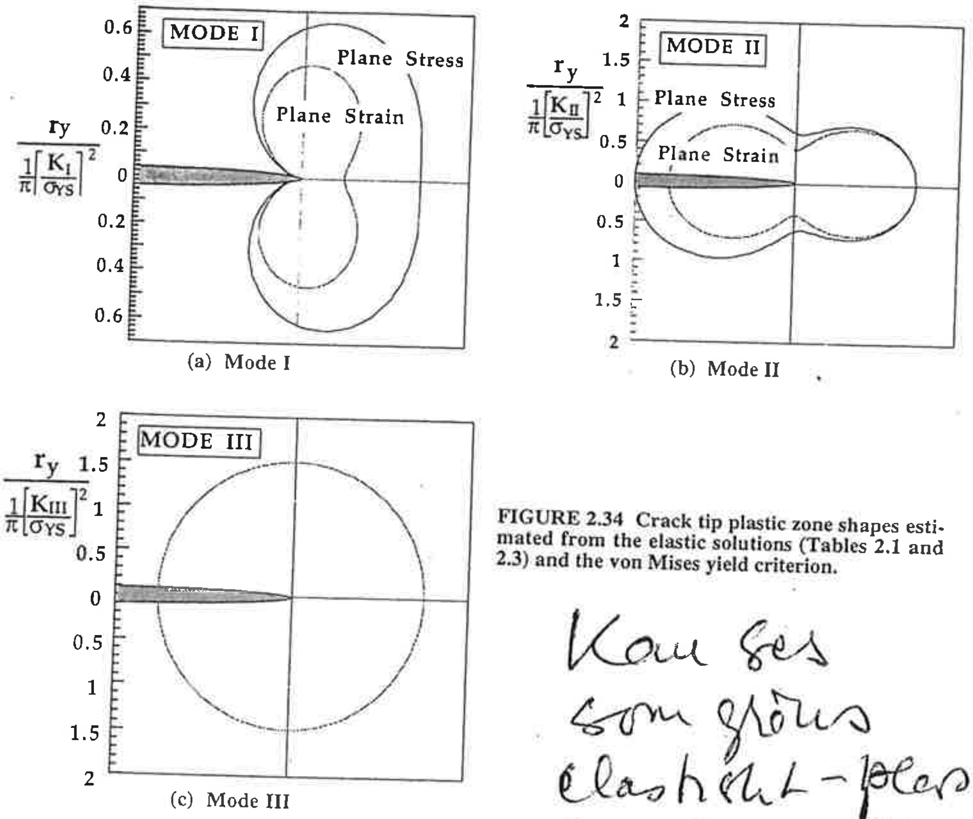


FIGURE 2.34 Crack tip plastic zone shapes estimated from the elastic solutions (Tables 2.1 and 2.3) and the von Mises yield criterion.

Kan ses
som gräns
elastiskt - plast
om $\sigma_{e_{Mises}} = \sigma_y$

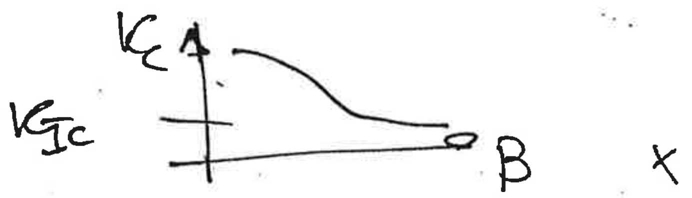
Note the significant difference in the size and shape of the Mode I plastic zones for plane stress and plane strain. The latter condition suppresses yielding, resulting in a smaller plastic zone for a given K_I value.

Equations (2.85a) and (2.85b) are not strictly correct because they are based on a purely elastic analysis. Recall Fig. 2.29, which schematically illustrates how crack tip plasticity causes stress redistribution, which is not taken into account in Fig. 2.34. The Irwin plasticity correction, which accounts for stress redistribution by means of an effective crack length, is also simplistic and not totally correct.

Figure 2.35 compares the plane strain plastic zone shape predicted from Eq. (2.85b) with a detailed elastic-plastic crack tip stress solution obtained from finite element analysis. The latter, which was published by Dodds, et al. [27], assumed a material with the following uniaxial stress-strain relationship:

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n \tag{2.86}$$

where ε_o, σ_o, α, and n are material constants. We will examine the above relationship in more detail in Chapter 3; for now it is sufficient to note that the exponent, n, charac-



singularity dominated zone is destroyed, and K no longer characterizes crack tip conditions. Thus the plastic zone must be embedded within the singularity dominated zone. In general, the singularity zone is small relative to in-plane length scales in the structure (see Example 2.6).

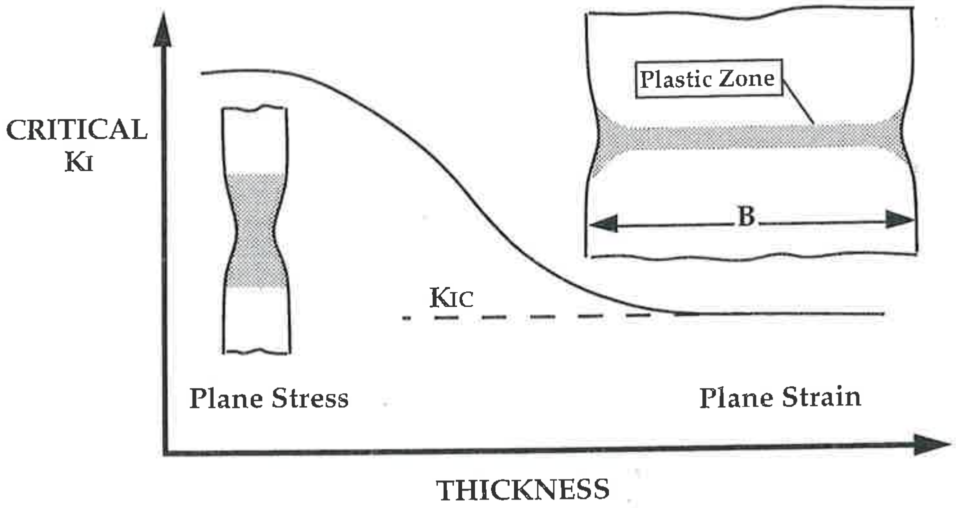


FIGURE 2.43 Effect of specimen thickness on Mode I fracture toughness.

EXAMPLE 2.7

Estimate the relative size of the singularity dominated zone ahead of a through crack in an infinite plate subject to remote uniaxial tension (Fig. 2.3). The full solution for the stresses on the crack plane ($\theta=0$) for this geometry are as follows (see Appendix 2.3.2):

$$\sigma_{yy} = \frac{\sigma(a+r)}{\sqrt{2ar + r^2}}$$

$$\sigma_{xx} = \frac{\sigma(a+r)}{\sqrt{2ar + r^2}} - \sigma$$

where σ is the remotely applied tensile stress. Also, estimate the value of K_I where the plane strain plastic zone engulfs the singularity dominated zone.

Solution: As $r \rightarrow 0$ both of the above relationships reduce to the expected result:

Temp